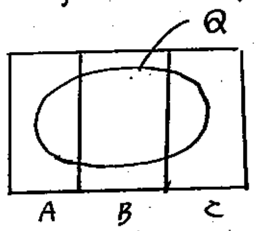
応数B,I 前期末 2019

試験答案用紙 番号 氏名

1. (1) A, B, C を J が 勝つ, 負ける, 引分ける事象とし, Q を Q選手がゴールを決める事象とする 仮定から



$P(A) = \frac{3}{5}$, $P(B) = \frac{3}{10}$, $P(C) = \frac{1}{10}$

$P(Q) = \frac{3}{10}$, $P_Q(A) = \frac{4}{5}$, $P_Q(B) = \frac{1}{10}$, $P_Q(C) = \frac{1}{10}$

[1] $P_A(Q) = \frac{P(A \cap Q)}{P(A)} = \frac{P(Q)P_Q(A)}{P(A)} = \frac{\frac{3}{10} \times \frac{4}{5}}{\frac{3}{5}} = \boxed{\frac{2}{5}}$

[2] $P_B(Q) = \frac{P(B \cap Q)}{P(B)} = \frac{P(Q)P_Q(B)}{P(B)} = \frac{\frac{3}{10} \times \frac{1}{10}}{\frac{3}{10}} = \boxed{\frac{1}{10}}$

[3] $P_C(Q) = \frac{P(C \cap Q)}{P(C)} = \frac{P(Q)P_Q(C)}{P(C)} = \frac{\frac{3}{10} \times \frac{1}{10}}{\frac{1}{10}} = \boxed{\frac{3}{10}}$

[4] $P_A(\bar{Q}) = \frac{P(A \cap \bar{Q})}{P(A)} = \frac{P(A) - P(A \cap Q)}{P(A)} = 1 - P_A(Q)$

$= 1 - \frac{2}{5} = \boxed{\frac{3}{5}}$

[5] [4]と同様にして $P_B(\bar{Q}) = 1 - P_B(Q) = 1 - \frac{1}{10} = \boxed{\frac{9}{10}}$

[6] $P_C(\bar{Q}) = 1 - P_C(Q) = 1 - \frac{3}{10} = \boxed{\frac{7}{10}}$

(2) [1] $\frac{40}{52} = \frac{10}{13}$ [2] $\frac{24}{52} = \frac{6}{13}$ [3] $\frac{13}{52} = \frac{1}{4}$

[4] $\frac{20}{52} = \frac{5}{13}$ [5] $\frac{6}{52} = \frac{3}{26}$ [6] $\frac{10}{52} = \frac{5}{26}$

[7] 与 $= P(A) + P(B) - P(A \cap B) = \frac{40}{52} + \frac{24}{52} - \frac{20}{52} = \frac{44}{52} = \frac{11}{13}$

[8] 与 $= \frac{24}{52} + \frac{13}{52} - \frac{6}{52} = \boxed{\frac{31}{52}}$

[9] $\frac{13}{52} + \frac{40}{52} - \frac{10}{52} = \boxed{\frac{43}{52}}$ [10] $\boxed{\frac{5}{52}}$

[11] 与式 $= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{1}{52}(40 + 24 + 13 - 20 - 6 - 10 + 5) = \frac{46}{52} = \frac{23}{26}$

2 (1) $E[7 - 3X] = 7 - 3E[X] = 7 - 15 = \boxed{-8}$

(2) [1] $E[X] = \frac{1}{6}\sum_{k=1}^{6} k = \frac{21}{6} = \boxed{\frac{7}{2}}$

[2] $E[X^2] = \frac{1}{6}\sum_{k=1}^{6} k^2 = \frac{1}{6} \cdot \frac{1}{6} \cdot 6 \cdot 7 \cdot 13 = \boxed{\frac{91}{6}}$

[3] $V[X] = E[X^2] - (E[X])^2 = \frac{91}{6} - \frac{49}{4} = \frac{35}{12} = \frac{70}{24}$

(3) [1] $a = 1 - \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$

[2] $E[X] = 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 3 \times \frac{1}{6} = \boxed{\frac{5}{3}}$

[3] $E[X^2] = 1 \times \frac{1}{2} + 4 \times \frac{1}{3} + 9 \times \frac{1}{6} = \frac{10}{3}$

$\therefore V[X] = E[X^2] - (E[X])^2 = \frac{10}{3} - \frac{25}{9} = \boxed{\frac{5}{9}}$

(4) [1] $E[X^2] = V[X] + (E[X])^2 = 3 + 25 = \boxed{28}$

[2] 与式 $= \frac{1}{\sqrt{6}}(3E[X] + 1) = \frac{1}{\sqrt{6}}(-15 + 1) = -\frac{14}{\sqrt{6}} = -\frac{7\sqrt{6}}{3}$

[3] 与式 $= \left(\frac{3}{\sqrt{6}}\right)^2 V[X] = \frac{3}{2} \times 3 = \boxed{\frac{9}{2}}$

(5) [1] 二項分布 $B(3, \frac{1}{3})$ に従う.

[2] $E[X] = 3 \times \frac{1}{3} = \boxed{1}$ [3] $V[X] = 3 \times \frac{1}{3} \times \frac{2}{3} = \boxed{\frac{2}{3}}$

[4] $E[X^2] = V[X] + (E[X])^2 = \frac{2}{3} + 1 = \boxed{\frac{5}{3}}$

(6) $n = 450$, $p = \frac{1}{300}$ より X は $B(450, \frac{1}{300})$ [1] に従う

よって $np = \frac{3}{2}$ [2] より $P_o(\frac{3}{2})$ で近似する.

$\therefore P(X = k) \fallingdotseq e^{-\frac{3}{2}} \frac{(\frac{3}{2})^k}{k!}$ ([3]: $-\frac{3}{2}$, [4]: $(\frac{3}{2})^k$)

よって $P(X \geqq 2) = 1 - P(X = 0) - P(X = 1) \fallingdotseq 1 - e^{-\frac{3}{2}}(1 + \frac{3}{2})$

$= 1 - \boxed{\frac{5}{2}}$ [5] $e^{-\frac{3}{2}} \fallingdotseq 0.442$ [6]

3. (1) [1] $\bar{x} = \frac{1}{10}(32 + 38 + \cdots + 73) = 54.7$

[2] $\bar{y} = \frac{1}{10}(125 + 128 + \cdots + 155) = 138.5$

[3] $\overline{x^2} = \frac{1}{10}(32^2 + 38^2 + \cdots + 73^2) = 3189.3$

[4] $\overline{y^2} = \frac{1}{10}(125^2 + 128^2 + \cdots + 155^2) = 19278.3$

[5] $\overline{xy} = \frac{1}{10}(32 \times 125 + \cdots + 73 \times 155) = 7707.1$

[6] $S_x^2 = \overline{x^2} - (\bar{x})^2 = 197.21$

[7] $S_y^2 = \overline{y^2} - (\bar{y})^2 = 96.05$

[8] $S_{xy} = \overline{xy} - \bar{x} \times \bar{y} = 131.15$

[9] $r = \frac{S_{xy}}{S_x S_y} = \frac{131.15}{\sqrt{197.21}\sqrt{96.05}} = 0.9529$

$y = ax + b$

[10] $a = \frac{S_{xy}}{S_x^2} = \frac{131.15}{197.21} = 0.6650$

[11] $b = \bar{y} - a\bar{x} = 102.1245$ (102.1230) OK

(2) [1] $\frac{1}{15}(6 + 4 + \cdots + 4) = 5.2 = \frac{78}{15} = \frac{26}{5}$

[2] $\frac{1}{15}(6^2 + 4^2 + \cdots + 4^2) = \frac{458}{15}$

よって 分散 $= \frac{458}{15} - \frac{676}{25} = \frac{2290 - 2028}{75} = \frac{262}{75} = 3.49\dot{3}$

小さい順に並べる

2, 3, 3, 4 | 4, 4, 5, 5, 6, 6, 6 | 6, 7, 8, 9

[3] $\boxed{5}$ [4] $\boxed{6}$ [5] $9 - 2 = \boxed{7}$

[6] $\boxed{4}$ [7] $\boxed{6}$ [8] $6 - 4 = \boxed{2}$

[9] $4 - 1.5 \times 2 = 1$, $6 + 1.5 \times 2 = 9$

より 外れ値は $\boxed{なし}$

4.(1) $M_z(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} exp\left(-\frac{z^2}{2} + tz\right) dz$

$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} exp\left\{-\frac{1}{2}(z-t)^2 + \frac{t^2}{2}\right\} dz$ (平方完成)

$= e^{\frac{t^2}{2}} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \sqrt{2}\, e^{-u^2} du$ (3) $\left(u = \frac{z-t}{\sqrt{2}}\right)$

$= e^{\frac{t^2}{2}} \frac{\sqrt{2}}{\sqrt{2\pi}}\sqrt{\pi} = e^{\frac{t^2}{2}}$ (4-1)

(2) $M_x(t) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2} + tx\right\} dx$

$= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} exp\left\{-\frac{z^2}{2} + t(\sigma z + \mu)\right\} \cdot \sigma dz$ $\left(z = \frac{x-\mu}{\sigma} \Rightarrow dx = \sigma dz,\ x = \sigma z + \mu\right)$

$= e^{\mu t}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} exp\left(-\frac{z^2}{2} + \sigma t z\right) dz$ $\left(z = \frac{x-\mu}{\sigma}\right)$

$= e^{\mu t} M_z(\sigma t) = exp\left(\frac{1}{2}\sigma^2 t^2 + \mu t\right)$ (4-1)

5(1) [1] $\bar{x} = 11$, $\overline{x^2} = 123$ $\therefore S_x^2 = 123 - 11^2 = 2$

[2] $\bar{y} = 26.2$, $\overline{y^2} = 1068.604$

[2] $S_y^2 = 1068.604 - 26.2^2 = 382.164$

[3] $\overline{xy} = 314.44$ $\therefore S_{xy} = 314.44 - 11 \times 26.2$

$S_{xy} = 26.24$

[4] $r_{xy} = \frac{S_{xy}}{S_x S_y} = 0.9491$

(2) z: 0.8061, 1.0413, 1.2810, 1.5314, 1.7817

[1] $S_z^2 = 0.09880$

[2] $\sum xz = 14.65956$, $\bar{z} = 1.2883$

$S_{xz} = 0.4882$

[3]

5. 2回カードを引くとき カードの数字の出方は $8^2 = 64$

1回目が 0か1なら 2回目は何でもよい, 1回目が 2なら 2回目は 0,1,2,3,4より $p_2 = \frac{8+8+5}{64} = \frac{21}{64}$ (1)

$n \geq 4$ のとき $x_{n-2} < \frac{8}{33}$ であって $x_{n-2} \neq 0.2424\cdots24$ となる確率は $p_{n-2} - \left(\frac{1}{8}\right)^{n-2}$ (2)

$x_{n-2} = 0.2424\cdots24$ となる確率は $\left(\frac{1}{8}\right)^{n-2}$ (3).

よってこのとき $x_n < \frac{8}{33}$ となる確率は x_2 のときと同じ考えより $\frac{21}{64}\left(\frac{1}{8}\right)^{n-2}$ (4) となる

$\therefore p_n = p_{n-2} - \left(\frac{1}{8}\right)^{n-2} + \frac{21}{64}\left(\frac{1}{8}\right)^{n-2} = p_{n-2} - 43\left(\frac{1}{8}\right)^n$ (5)

$p_n - p_{n-2} = -43\left(\frac{1}{8}\right)^n$ $\therefore \sum_{k=2}^{m}\{p_{2k} - p_{2(k-1)}\} = p_{2m} - p_2$ であり

$\sum_{k=2}^{m}\{p_{2k} - p_{2(k-1)}\} = \sum_{k=2}^{m}\left(-43\left(\frac{1}{8}\right)^{2k}\right)$ (6) $= -43\frac{\left(\frac{1}{8}\right)^4\left\{1 - \left(\frac{1}{8}\right)^{2(m-1)}\right\}}{1 - \left(\frac{1}{8}\right)^2}$

$= -\frac{43}{63}\left(\frac{1}{64} - \left(\frac{1}{8}\right)^{2m}\right)$ (7) $\therefore p_{2m} = \frac{21}{64} - \frac{43}{63}\left(\frac{1}{64} - \left(\frac{1}{8}\right)^{2m}\right)$

$= \frac{20}{63} + \frac{43}{63}\left(\frac{1}{8}\right)^{2m}$ (8) (7)