SC1 前末 2017

20~~16~~17年度試験答案用紙

番号　　氏名

1. (1) [1] $|1+i| = \sqrt{1^2+1^2} = \boxed{\sqrt{2}}$

[2] $|-2+3i| = \sqrt{(-2)^2+3^2} = \sqrt{4+9} = \boxed{\sqrt{13}}$

[3] $\left|\dfrac{1}{3-\sqrt{3}i}\right| = \dfrac{1}{\sqrt{3^2+(-\sqrt{3})^2}} = \dfrac{1}{\sqrt{9+3}} = \boxed{\dfrac{1}{2\sqrt{3}}}$

(2) [1] 与式 $= \dfrac{a(a-2b)+2ab}{a^2-4b^2} = \boxed{\dfrac{a^2}{a^2-4b^2}}$

[2] 与式 $= \dfrac{x(x+1)+1-x}{x+1-x} = \boxed{x^2+1}$

[3] 与式 $= \sqrt{3}(\sqrt{2}-1)(\sqrt{2}+1) = \sqrt{3}(2-1) = \boxed{\sqrt{3}}$

[4] 与式 $= \dfrac{\sqrt{2}}{\sqrt{2}+1}\cdot\dfrac{1}{\sqrt{2}(\sqrt{2}+1)} = \dfrac{1}{(\sqrt{2}+1)^2} = \boxed{\dfrac{1}{3+2\sqrt{2}}}$

$= \dfrac{3-2\sqrt{2}}{(3+2\sqrt{2})(3-2\sqrt{2})} = \dfrac{3-2\sqrt{2}}{9-8} = \boxed{3-2\sqrt{2}}$ 1点 ↑ これは× とする

(3) [1] 与式 $= \sqrt{5}-2+5-\sqrt{5} = \boxed{3}$

[2] 与式 $= |(2\sqrt{6}-5)(2\sqrt{6}+5)| = |24-25| = \boxed{1}$

[3] 与式 $= \dfrac{3\sqrt{3}}{\sqrt{3}i} = \dfrac{3}{i} = \boxed{-3i}$ → $\dfrac{3}{i}$ は×とする

[4] 与式 $= \sqrt{2}i\cdot 3\sqrt{2}i = \boxed{-6}$

(4) [1] 与式 $= (1+2i)^2 = 1+4i+(2i)^2 = \boxed{-3+4i}$

[2] 与式 $= 1+2i+1-2i = \boxed{2}$

(5) [1] 与式 $= \sqrt{(\sqrt{2}-1)^2} = \boxed{\sqrt{2}-1}$

[2] 与式 $= \sqrt{2+2\sqrt{\dfrac{3}{4}}} = \sqrt{\left(\sqrt{\dfrac{3}{2}}+\dfrac{1}{\sqrt{2}}\right)^2} = \boxed{\dfrac{\sqrt{3}+1}{\sqrt{2}} = \dfrac{\sqrt{6}+\sqrt{2}}{2}}$

[3] 与式 $= \sqrt{27-2\sqrt{50}} = \sqrt{(5-\sqrt{2})^2} = \boxed{5-\sqrt{2}}$

(6) 与式 $= \sqrt{(\sqrt{x}-1)^2} = |\sqrt{x}-1| = \boxed{1-\sqrt{x}}$ ($\because$ $\sqrt{x}-1 \leqq 0$)

(7) $\dfrac{8}{\sqrt{6-2\sqrt{5}}} = \dfrac{8}{\sqrt{5}-1} = \dfrac{8}{4}(\sqrt{5}+1) = \boxed{2\sqrt{5}+2}$ [1]

$2 < \sqrt{5} < \dfrac{5}{2}$ より $6 < 2\sqrt{5}+2 < 7$ $\therefore$ $a = \boxed{6}$ [2]

$b = \boxed{2\sqrt{5}-4}$ [3] $\therefore$ $\dfrac{1}{a}+\dfrac{1}{b} = \dfrac{1}{6}+\dfrac{1}{2(\sqrt{5}-2)}$

$= \dfrac{1}{6}+\dfrac{1}{2}(\sqrt{5}+2) = \boxed{\dfrac{7+3\sqrt{5}}{6}}$ [4]

2. (1) [1] $x = 5\pm\sqrt{25-25} = \boxed{5}$ (2重解)

[2] $x = \dfrac{-3\pm\sqrt{9-4}}{2} = \boxed{\dfrac{-3\pm\sqrt{5}}{2}}$

[3] $x = \dfrac{5\pm\sqrt{25-28}}{2} = \boxed{\dfrac{5\pm\sqrt{3}i}{2}}$

(2) [1] $D = 4-20 = \boxed{-16}$ [2] $D = 1-4\cdot\dfrac{1}{4} = \boxed{0}$

(3) 判別式をDとする. $\dfrac{D}{4} = k^2-(2-k) = k^2+k-2 = (k+2)(k-1)$

$D=0$ より $k = \boxed{-2, 1}$

(4) $\alpha+\beta = -\dfrac{1}{2}$, $\alpha\beta = \dfrac{1}{2}$ だから $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = \dfrac{1}{4}-1$

$= \boxed{-\dfrac{3}{4}}$

(5) [1] $3x^2-7x+5=0$ を解いて $x = \dfrac{7\pm\sqrt{49-60}}{6} = \dfrac{7\pm\sqrt{11}i}{6}$

$\therefore$ $\boxed{3\left(x-\dfrac{7+\sqrt{11}i}{6}\right)\left(x-\dfrac{7-\sqrt{11}i}{6}\right)}$

[2] $2x^2+3x-1=0$ を解いて $x = \dfrac{-3\pm\sqrt{9+8}}{4} = \dfrac{-3\pm\sqrt{17}}{4}$

$\therefore$ $\boxed{2\left(x+\dfrac{3+\sqrt{17}}{4}\right)\left(x+\dfrac{3-\sqrt{17}}{4}\right)}$

(6) $X = x^2$ とおけば $2X^2-X-1=0$, $(2X+1)(X-1)=0$

$x^2 = -\dfrac{1}{2}, 1$ $\therefore$ $x = \boxed{\pm 1, \pm\dfrac{i}{\sqrt{2}}}$

(7) $P(1) = 1-4+10-17+10 = 0$ 組立除法

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 10 & -17 & 10 \\ & & 1 & -3 & 7 & -10 \\ \hline & 1 & -3 & 7 & -10 & 0 \end{array}$$

$\therefore$ $P(x) = (x-1)(\boxed{x^3-3x^2+7x-10})$ [1]

$Q(x) = x^3-3x^2+7x-10$ → $Q(2) = 8-12+14-10 = 0$

$\therefore$ $Q(x) = (x-2)(\boxed{x^2-x+5})$ [2] 以上より

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 7 & -10 \\ & & 2 & -2 & 10 \\ \hline & 1 & -1 & 5 & 0 \end{array}$$

$P(x)=0$ の解は $x = \boxed{1, 2, \dfrac{1\pm\sqrt{19}i}{2}}$ [3]

(8) [1] $x \geqq 0$ のとき $2x = x+2$ より $x=2$, $x<0$ のとき

$-2x = x+2$ より $x = -\dfrac{2}{3}$ $\therefore$ $x = \boxed{-\dfrac{2}{3}, 2}$

[2] 両辺に x^2-3x+2 をかけて $x(x-1)-4(x-2) = x+3$

$x^2-6x+5=0$, $(x-1)(x-5)=0$ $x=1$ は無縁解

$x = \boxed{5}$

[3] 左辺 $\geqq 0$ より $x \geqq 3$. 両辺を2乗して $x+3 = x^2-6x+9$

$x^2-7x+6=0$, $(x-1)(x-6)=0$ $\therefore$ $x = \boxed{6}$

3. (1) $\bar{\alpha} = \boxed{a-bi}$

(2) $\alpha+\beta = a+c+(b+d)i$ より $\overline{\alpha+\beta} = \boxed{a+c-(b+d)i}$

(3) $|\alpha|^2 = \boxed{a^2+b^2}$

(4) 与式 $= (a+bi)(c-di)+(a-bi)(c+di)$

$= ac+bd+(bc-ad)i+ac+bd+(ad-bc)i$

$= \boxed{2(ac+bd)}$

(5) 与式 $= a^2+b^2+2(ac+bd)+c^2+d^2 = (\boxed{a+c})^2+(\boxed{b+d})^2$ [1] [2] 順不同

5段

4. (1) $\sqrt{1+x} = \sqrt{1+\dfrac{2a}{1+a^2}} = \sqrt{\dfrac{(1+a)^2}{1+a^2}} = \dfrac{|1+a|}{\sqrt{1+a^2}}$

$-1 \leqq a \leqq 1$ より $a+1 \geqq 0$ よって 与式 $= \boxed{\dfrac{1+a}{\sqrt{1+a^2}}}$

(2) $\sqrt{1-x} = \sqrt{1-\dfrac{2a}{1+a^2}} = \sqrt{\dfrac{(1-a)^2}{1+a^2}} = \dfrac{|1-a|}{\sqrt{1+a^2}}$

$= \dfrac{1-a}{\sqrt{1+a^2}}$ ($\because$ $-1 \leqq a \leqq 1$ より $1-a \geqq 0$)

(3) $\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} = \dfrac{\dfrac{1+a+a-1}{\sqrt{1+a^2}}}{\dfrac{1+a+1-a}{\sqrt{1+a^2}}} = \dfrac{2a}{2} = a$

3段

5. $x^4+2x^2+4 = x^4+4x^2+4-2x^2 = (x^2+2)^2-(\sqrt{2}x)^2$ より

方程式は $(x^2+\sqrt{2}x+2)(x^2-\sqrt{2}x+2) = 0$ となる.

(i) $x^2+\sqrt{2}x+2=0$ を解いて $x = \dfrac{-\sqrt{2}\pm\sqrt{2-8}}{2} = \dfrac{-\sqrt{2}\pm\sqrt{6}i}{2}$

(ii) $x^2-\sqrt{2}x+2=0$ を解いて $x = \dfrac{\sqrt{2}\pm\sqrt{6}i}{2}$

以上より $x = \pm\dfrac{\sqrt{2}+\sqrt{6}i}{2}$, $\pm\dfrac{\sqrt{2}-\sqrt{6}i}{2}$

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6. $x^2 = (a+bi)^2 = \boxed{a^2-b^2}+\boxed{2ab}i$ (1) (2)

$\therefore$ $\begin{cases} a^2-b^2 = -1 & \cdots ① \\ 2ab = \sqrt{3} & \cdots ② \end{cases}$ ② から $b = \dfrac{\sqrt{3}}{2a}$ これと ① から

$a^2-\dfrac{3}{4a^2}+1=0$ $4a^4+4a^2-3=0$ $(2a^2-1)(2a^2+3)=0$

a は実数だから $a = \pm\boxed{\dfrac{\sqrt{2}}{2}}$ $\therefore$ $b = \pm\boxed{\dfrac{\sqrt{6}}{2}}$ (複号同順)

(3) (4)

十

7. $\begin{cases} x+4y-2z+4w=12 & ②-2×①,\ ③-3×① \\ -5y+10z-10w=\boxed{-15} \cdots ②' & (4-4×①)÷3 \\ -5y+9z-10w=\boxed{-18} \cdots ③' \\ -5y+5z-7w=\boxed{-15} \cdots ④' \end{cases}$

$\rightarrow \begin{cases} x+4y-2z+4w=12 \cdots ① \\ -5y+10z-10w=-15 \cdots ②' \\ -z=\boxed{-3} \cdots ③'' & ③'-②',\ ④'-②' \\ -5z+3w=\boxed{0} \cdots ④'' \end{cases}$

$\therefore z=\boxed{3}$ これと ④'' から $w=\boxed{5}$

$\therefore 5y=10z-10w+15=-20+15=-5 \quad \therefore y=\boxed{-1}$

$x=12-4y+2z-4w=12+4+6-20=\boxed{2}$ 9

(1) -15 (2) -18 (3) -15 (4) -3 (5) 0 (6) 3
(7) 5 (8) -1 (9) 2

5
配2

8. $x^2-a^2=(2\sqrt{a-1})^2-a^2=4(a-1)-a^2$

$=-(a^2-4a+4)=-(a-2)^2$

$\therefore \sqrt{x^2-a^2}=\sqrt{-(a-2)^2}=\sqrt{(a-2)^2}\,i$

$=\begin{cases} (a-2)i & (a \geqq 2) \\ (2-a)i & (1 \leqq a < 2) \end{cases}$ 4

9. Hint より $\begin{cases} x+y+z=25 \cdots ① \\ xy=50 \cdots ② \\ z^2=x^2+y^2 \cdots ③ \end{cases}$ を解く。①から

$z=25-(x+y)$ となり、これと ②, ③ から $x+y=\dfrac{29}{2} \cdots ④$

④と② および 2次方程式の解と係数の関係から x, y は

2次方程式 $X^2-\dfrac{29}{2}X+50=0$ の解で、これを解いて

$X=\dfrac{29-\sqrt{41}}{4},\ \dfrac{29+\sqrt{41}}{4}$ 斜辺は

$z^2=(x+y)^2-2xy=\left(\dfrac{29}{2}\right)^2-100=\dfrac{441}{4}$

$\therefore z=\dfrac{21}{2}$

よって 3辺は $\dfrac{29\pm\sqrt{41}}{4},\ \dfrac{21}{2}$ 8

(詳細)

③: $\{25-(x+y)\}^2=x^2+y^2$

$25^2-50(x+y)+(x+y)^2=x^2+y^2$

$\therefore 50(x+y)=25^2+2xy \underset{②}{=} 25^2+100$

$x+y=\dfrac{725}{50}=\dfrac{29}{2}$