

1. (1) [1] R = P(2) = 8 + 4a - 8 + 3 = 4a + 3

[2] S = P(-3) = -27 + 9a + 12 + 3 = 9a - 12

[3] 4a + 3 = 9a - 12 ⇒ a = 3

(2) 問題の整式 P(x) を表す。P(-1) = -1 + 2 + 5 - 6 = 0 かつ P(x) は x+1 に因数分解される。

P(x) = (x+1)(x^2 + x - 6) = (x+1)(x+3)(x-2)

(3) 因数分解の式を求めよ。

(x-3)(x+5), (x-3)(2x+1), (x-3)^2 など

[1] L.C.M. = (x-3)^2(x+5)(2x+1) = 2x^4 - x^3 - 43x^2 + 69x + 45

[2] G.C.M. = x-3

(4) A = (2x+1)Q + R = (x + 1/2)2Q + R ⇒ 2Q = 6x + 2 ⇒ Q = 3x + 1, R = -2

∴ A = (2x+1)(3x+1) - 2

(5) [1] 余り = A(-2/3) = 3(-2/3)^2 - 2/3 + 1 = 4/3 - 2/3 + 1 = 5/3

[2] 余り = A(2) = 2^3 + 2^3 + 6 + 5 = 16 + 11 = 27

(6) P(x) = 2x^3 - 5x^2 - x + a かつ P(-3/2) = 0 ならば

P(-3/2) = 2(-3/2)^3 - 5(-3/2)^2 - 3/2 + a = -27/4 - 45/4 + 3/2 + a = -33/2 + a = 0 ⇒ a = 33/2

(7) [1] 式 = (x-10)(x+2)

[2] 3x^2 - 2 → 8, 4x - 1 → -3, 12 - 2 → 5 ⇒ 式 = (3x+2y)(4x-y)

[3] 式 = x^2(4x-9y)

[4] 式 = x^2 + (y+2)x - (2y^2+5y+3)

= x^2 + (y+2)x - (2y+3)(y+1) = (x-y-1)(x+2y+3)

[5] 式 = a(b-c) - (b-c) = (a-1)(b-c)

2. (1) [1] 式 = 2x^2 - 9 + x = 2x^2 + x - 9

[2] 式 = -3x^3 + x^3 - 4x = -4x - 3x^2 + x^3

(2) [1] A+B = (3x^3 - 2x^2 + x) + (2x^3 - 2x + 4) = 5x^3 - 2x^2 - x + 4

[2] A-B = (x - 2x^2 + 3x^3) - (4 - 2x + 2x^3) = -4 + 3x - 2x^2 + x^3

(3) [1] 式 = 2x^2 + 5xy - 3y^2

[2] 式 = -8a^3b^9, [3] 式 = 4x^2 + 12x + 9

[4] 式 = a^3 + 3^3 = a^3 + 27

[5] 式 = (a^2+1)+a / (a^2+1)-a = (a^2+1)^2 - a^2 = a^4 + a^2 + 1

[6] 式 = a^2b^2(-2a^3b) = -2a^5b^3

[7] 式 = (2x)^3 + 3(2x)^2·3y + 3·2x(3y)^2 + (3y)^3 = 8x^3 + 36x^2y + 54xy^2 + 27y^3

[8] 式 = x^2 + 4y^2 + 9 - 4xy - 6x + 12y = x^2 - 4xy + 4y^2 - 6x + 12y + 9 = (x-2y)^2 - 6x + 12y + 9

(4) [1] a^2b - ab^2 = ab(a-b), a^3 - b^3 = (a-b)(a^2 + ab + b^2) ⇒ G.C.M. = (a-b), L.C.M. = ab(a-b)(a^2 + ab + b^2) = a^4b - ab^4

[2] G.C.M. = ac, L.C.M. = a^4b^3c^3d

(5) 式 = (a^3-1)/a^2 * a/(a-1) = (a-1)(a^2+a+1)/(a(a-1)) = (a^2+a+1)/a

(6) [1] 式 = (3xy^3)/(2z^3)

[2] 式 = x(y^2-x^2)/(x^2y(y+x)) = (y+x)(y-x)/(xy(y+x)) = (y-x)/xy

(7) (1) 式 = a/(a+2b) + 2ab/((a+2b)(a-2b)) = (a(a-2b) + 2ab)/((a+2b)(a-2b)) = a^2/(a^2-4b^2)

[2] 式 = (x/2)(x/1) * x/3 * x/(x+2) / (x(x-3) * (x+1)(x/2)) = 1

3. P(1) = 1 - 4 - 5 - 2 + 10 = 0 かつ P(x) は (x-1) に

割り切れる。P(x) ÷ (x-1) = (x^3 - 3x^2 - 8x - 10) / (x-1) = x^2 - 2x - 10 + (10/x-1)

Q(5) = 125 - 75 - 40 - 10 = 0 かつ Q(x) は (x-5) に割り切れる。Q(x) ÷ (x-5) = (x^2 + 2x + 2) / (x-5) = x + 7 + (17/x-5)

∴ P(x) = (x-1)(x-5)(x^2 + 2x + 2) - 16 (1) x-1 (2) x^3 - 3x^2 - 8x - 10 (3) 5 (4) x-5 (5) x^2 + 2x + 2

4. (1) 式 = 4x^4 + 4x^2 + 1 - x^2 = (2x^2+1)^2 - x^2 = (2x^2+x+1)(2x^2-x+1)

(2) 式 = (a+b)(a^2-ab+b^2) + ab(a+b) = (a+b)(a^2+b^2)

(3) X = x^3 とおくと 式 = X^2 - 9X + 8 = (X-1)(X-8) = (x^3-1)(x^3-8) = (x-1)(x-2)(x^2+x+1)(x^2+2x+4)

5. A ÷ B ⇒ 1 - 3 2 / 1 - 2 - 1 4 ⇒ 商 = x+1, 余り = 2

B = (x-1)(x-2), D = (x-1)(x-3) ならば B と D の最小公倍数は (x-1)(x-2)(x-3) ⇒ A/B - C/D = (2(x-3) - 3(x-2)) / ((x-1)(x-2)(x-3)) = (-x-6) / ((x-1)(x-2)(x-3))

6. 仮定から P(x) = (x^2+x+1)Q(x) + x^2+1 とおくと P(x) は x^2+x+1 で割り切れる。P(x) ÷ (x^2+x+1) の商は

1 1 1 / 1 0 0 0 1 ⇒ 商 = x+1 (3)

~~A = ...~~

AB = LG

4) $A = (x-6)(x+1)$ $\therefore AB = (x+1)(x^3 - 10x^2 + 19x + 30) = LG$
 (L: 最小公倍数, G: 最大公因数)

だから L.C.M は $x-6$ 3, 因数に x 5, 割り地 x 3 3 2

$$x^3 - 10x^2 + 19x + 30$$

$$= (x-6)(x^2 - 4x - 5)$$

$$\therefore B = (x+1)(x-5) = x^2 - 4x - 5$$

$$\begin{array}{r|rrrr} 6 & 1 & -10 & 19 & 30 \\ & & 6 & -24 & -30 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

(5)

8. $C \div G$

$$\begin{array}{r|rrrrrr} 1 & 1 & 2 & -3 & -3 & 2 & 1 \\ & & 1 & 3 & 0 & -3 & -1 \end{array}$$

$$= (x^4 + 3x^3 - 3x - 1)$$

$D \div 4$

$$= (x^3 + 4x^2 + 4x + 1) \quad \begin{array}{l} (2) \quad x = -1 \quad \therefore \\ E \quad \rightarrow \quad -1 + 4 - 4 + 1 = 0 \end{array}$$

$$\therefore a = -1$$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 4 & 1 \\ & & -1 & -3 & -1 \\ \hline & 1 & 3 & 1 & 0 \end{array}$$

$$\therefore E \div (x+1)$$

$$= (x^2 + 3x + 1)$$

$$\therefore AB = (x-1)^2(x+1)(x^2+3x+1), \text{ AB の次数は } 5$$

$$P = (x+1), \quad Q = (x^2+3x+1)$$

$$\therefore A = (x-1)(x+1) = x^2 - 1$$

$$B = (x-1)(x^2+3x+1) = x^3 + 2x^2 - 2x - 1$$

9. $x^3 + y^3 + z^3 - 3xyz = (x+y)^3 + z^3 - 3x^2y - 3xy^2 - 3xyz$

$$= (x+y+z) \{ (x+y)^2 - z(x+y) + z^2 \} - 3xy(x+y+z)$$

$$= (x+y+z)(x^2 + y^2 + z^2 + 2xy - zx - yz - 3xy)$$

$$= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

(4)

7. $G = x+1, L = x^3 - 10x^2 + 19x + 30$ と表す.

$$AB = LG \quad \therefore A = x^2 - 5x - 6 = (x-6)(x+1) = (x-6)G \text{ となる}$$

$$B = \frac{LG}{A} = \frac{LG}{(x-6)G} = \frac{L}{x-6} \rightarrow \begin{array}{r|rrrr} 6 & 1 & -10 & 19 & 30 \\ & & 6 & -24 & -30 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$$\therefore B = x^2 - 4x - 5$$

* Point. B は L を $x-6$ で割った商に等しい。

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \quad (1)$$

$$x^3 + y^3 = (x+y)^3 - 3x^2y - 3xy^2$$

$$(x+y)^3 + z^3 = a^3 + z^3 \quad (a = x+y)$$

$$= (a+z)(a^2 - az + z^2)$$