MS2 微積2 学年末. 2018

試験答案用紙　　番号　　氏名

1. $I=\int x^{-1/2}(x^2+1)^{-5/4}dx$ となる　$\frac{-1/2+1}{2}-\frac54=\frac14-\frac54=-1$ [3]

$t=(1+x^{-2})^{1/4}$ と置換する. $dt=\frac14(1+x^{-2})^{-3/4}\cdot(-2x^{-3})$

$dt=-\frac12\left(\frac{x^2+1}{x^2}\right)^{-3/4}\cdot x^{-3}dx=-\frac12x^{-3/2}(x^2+1)^{-3/4}dx$ [4][5]

$\therefore x^{-1/2}(x^2+1)^{-5/4}\cdot x^{3/2}(x^2+1)^{3/4}=x(x^2+1)^{-1/2}$ [6][7]

一方 $t^2=\left(\frac{x^2+1}{x^2}\right)^{1/2}=x^{-1}(x^2+1)^{1/2}$ となる　よって $=\frac{1}{t^2}=t^{-2}$ となる [8]

$\therefore I=\int\left(\frac{-2}{t^2}\right)dt=\left(\frac{2}{t}\right)=2\left(\frac{x^2+1}{x^2}\right)^{-1/4}=\frac{2\sqrt{x}}{\sqrt[4]{x^2+1}}$ [9][10][11]

6段

---

2. $y'=-y\cos x$ より $\int\frac{dy}{y}=-\int\cos x\,dx+C$

$\therefore \log|y|=-\sin x+C$　$\therefore y=Ce^{-\sin x}$ (1)

$u=u(x)$ として $y=ue^{-\sin x}$ を① に代入すれば

$u'e^{-\sin x}=\cos x\sin x$ より $u'=e^{\sin x}\cos x\sin x$ (2)

$u=\int e^{\sin x}\sin x\cos x\,dx+C$, $t=\sin x$ と置換すれば

$u=\int te^t dt=te^t-e^t=e^t(t-1)$ (3)(4)

$\therefore u=e^{\sin x}(\sin x-1)+C$ (5)

$\therefore y=ue^{-\sin x}=Ce^{-\sin x}+\sin x-1$ (6)

4段　5段

---

3. $u=x-3y$ …② とおくと $u'=1-3y'$, また

① は $y'=\frac{2u+7}{u+4}$ となるから $u'=1-\frac{6u+21}{u+4}=\frac{-5u-17}{u+4}$

$\frac{u+4}{5u+17}u'=-1$　$\therefore\int\frac{u+4}{5u+17}du=-x+C$

左辺 $=\int\left(\frac15+\frac35\cdot\frac{1}{5u+17}\right)du=\frac15u+\frac{3}{25}\log|5u+17|$

$e^{5u}(5u+17)^3=Ce^{-25x}$　② より $\therefore e^{30x-15y}(5x-15y+17)^3=C$

5行

($\log|5x-15y+17|+10x-5y=C$ でも ok)

$5u+17=0$, すなわち $5x-15y+17=0$ も解である.

---

4 (1) $\int x^{3/2}dx=\frac25x^{5/2}=\frac25\sqrt{x^5}=\frac25x^2\sqrt{x}$

(2) $\log|x|$　(3) e^x　(4) $-\cos x$

(5) $\sin x$　(6) $\tan x$　(7) $-\cot x$

(8) $\sin^{-1}\frac{x}{\sqrt5}$　(9) $\frac12\tan^{-1}\frac{x}{2}$

(10) $\log|x+\sqrt{x^2-3}|$

(11) $\frac{1}{45}e^{-6x}(-6\sin3x-3\cos3x)$　1点

$=-\frac{1}{15}e^{-6x}(2\sin3x+\cos3x)$

(12) $\frac12\left(x\sqrt{6-x^2}+6\sin^{-1}\frac{x}{\sqrt6}\right)$

(13) $\frac14\log\left|\frac{x-2}{x+2}\right|$

(14) $\frac12\left(x\sqrt{x^2-6}-6\log|x+\sqrt{x^2-6}|\right)$

---

5. (1) $I_1=\int_0^\infty\frac{x}{(1+x^2)^2}dx=\frac12\int_0^\infty\frac{2x}{(1+x^2)^2}dx$

$=\frac12\left[-\frac{1}{1+x^2}\right]_0^\infty=\frac12$

(2) $\int\frac{x}{(1+x^2)^{n+1}}dx=-\frac{1}{2n(1+x^2)^n}$ [1]

$n\geq2$

$I_n=\int_0^\infty x^{2n-2}\cdot\frac{x}{(1+x^2)^{n+1}}dx=\left[-\frac{x^{2n-2}}{2m(1+x^2)^n}\right]_0^\infty+\int_0^\infty\frac{(2m-2)x^{2n-3}}{2m(1+x^2)^n}dx$

$=\frac{n-1}{m}I_{m-1}$ [3]　$\left(1-\frac1m\right)$ この形は( )が必要

[2] $\frac{(m-1)x^{2n-3}}{m(1+x^2)^n}$

$\therefore I_m=\frac{n-1}{m}\cdot\frac{n-2}{n-1}\cdots\cdot\frac23\cdot\frac12I_1=\frac1mI_1$ [4]

$I_1=\frac12$ となる　$I_m=\frac{1}{2m}$ [5]

4段　5行

---

6. $\int\cos x\sin^4x\,dx=\frac15\sin^5x$ (1)

$I_{2,4}=\int\cos x\cdot\cos x\sin^4x\,dx$

$=\frac15\sin^5x\cos x+\int\frac15\sin^6x\,dx$ [2][3]

$=\frac15\sin^5x\cos x+\frac15\int(1-\cos^2x)\sin^4x\,dx$

$=\frac15\int\sin^4x\,dx-\frac15I_{2,4}$

$\therefore I_{2,4}=\frac16\sin^5x\cos x+\frac16\int\sin^4x\,dx$ …① (4)(5)(6)

$\int\sin^4x\,dx=-\cos x\sin^3x+3\int\cos^2x\sin^2x\,dx$ (7)

$=-\cos x\sin^3x+3\int\sin^2x\,dx-3\int\sin^4x\,dx$ [8](9)

$\therefore\int\sin^4x\,dx=-\frac14\sin^3x\cos x+\frac34\int\sin^2x\,dx$ …② (10)(11)

$\int\sin^2x\,dx=\frac12\int(1-\cos2x)dx=\frac12x-\frac14\sin2x=\frac12(x-\sin x\cos x)$ (12)

$\therefore\int\sin^4x\,dx=-\frac14\sin^3x\cos x-\frac38\sin x\cos x+\frac38x$

$\therefore I_{2,4}=\frac16\sin^5x\cos x-\frac{1}{24}\sin^3x\cos x-\frac{1}{16}\sin x\cos x+\frac{1}{16}x$

$=\frac{1}{48}\cos x(8\sin^5x-2\sin^3x-3\sin x)+\frac{1}{16}x$

$=\frac{1}{48}\sin x\cos x(8\sin^4x-2\sin^2x-3)+\frac{1}{16}x$ (13) [14]

まとめ

[1] $\frac15\sin^5x$　[2] $\frac15$

[3] 6　(4) $\sin^4x$

(5) ~~$\frac16\sin^5x\cos x$~~

[6] $\frac16$　(7) $\sin^3x$

[8] 3　(9) $\sin^2x$

(10) $-\frac14\sin^3x\cos x$　[11] $\frac34$

(12) $\frac12\left(x-\frac12\sin2x\right)=\frac12(x-\sin x\cos x)$

(13) $8\sin^4x-2\sin^2x-3$

[14] $\frac{1}{16}$

7. $\alpha = 3/2$ とれば $l = \lim_{x\to 3-0} (3-x)^{3/2} \dfrac{\sqrt{1+x^3}}{(3-x)^2}$

$l = \lim_{x\to 3-0} \dfrac{\sqrt{1+x^3}}{\sqrt{3-x}} = \infty$ より 発散する. (2点)

※ $\alpha = 2$ とし $l = \sqrt{28} > 0$ で 発散としてもよい.

---

8. I において $x = \dfrac{\pi}{4} - t$ と置換すれば $dx = -dt$

$$\begin{array}{c|c} x & 0 \to \pi/4 \\ \hline t & \pi/4 \to 0 \end{array}, \quad 1+\tan x = 1 + \tan\left(\frac{\pi}{4} - t\right) = 1 + \frac{1-\tan t}{1+\tan t}$$

$\therefore 1 + \tan x = \dfrac{2}{1+\tan t}$ なので

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan t}\right) dt = \int_0^{\pi/4} (\log 2 - \log(1+\tan t))\, dt$$

$= \dfrac{\pi}{4}\log 2 - I \qquad \therefore I = \dfrac{\pi}{8}\log 2$ (5点)

---

9. $dx = \left(-\dfrac{1}{(t+1)^2}\right) dt$ (1), $\lim_{x\to +0} t = \lim_{x\to +0}\left(\dfrac{1}{x} - 1\right) = \boxed{\infty}$ (2)

$1 - x = 1 - \dfrac{1}{t+1} = \dfrac{t}{t+1}$ (3)

$$\therefore B(p,q) = \int_\infty^0 \frac{1}{(t+1)^{p-1}}\left(\frac{t}{t+1}\right)^{q-1}\cdot\left(-\frac{1}{(t+1)^2}\right) dt$$

$$= \int_0^\infty \frac{t^{q-1}}{(t+1)^{\boxed{p+q}}}\, dt$$ (4)

(4) $p+q$ (5) $q-1$

また $x = \cos^2\theta \ (0 \leqq \theta \leqq \pi/2) \to dx = (-2\cos\theta\sin\theta)\, d\theta$, (6)

$\lim_{x\to +0} \theta = \boxed{\pi/2}$ [7], $\lim_{x\to 1-0} \theta = \boxed{0}$ [8]

$$\therefore B(p,q) = \int_{\pi/2}^0 (\cos^2\theta)^{p-1}(\sin^2\theta)^{q-1}\cdot(-2\cos\theta\sin\theta)\, d\theta$$

$$= \boxed{2}\int_0^{\pi/2} (\cos\theta)^{\boxed{2p-1}}(\sin\theta)^{\boxed{2q-1}}\, d\theta$$ [9] (10) (11)

6段

(10) $2p-1$ (11) $2q-1$

---

10. $x = \sec\theta \ (0 \leqq \theta < \pi/2,\ \pi \leqq \theta < 3\pi/2)$ と置換すると

$dx = \tan\theta\sec\theta\, d\theta$, $\sqrt{x^2-1} = |\tan\theta| = \tan\theta$ (θの範囲)

$$\therefore I = \int \frac{\tan\theta}{\sec^4\theta}\cdot\tan\theta\sec\theta\, d\theta = \int \sin^2\theta\cos\theta\, d\theta$$

$= \dfrac{1}{3}\sin^3\theta$, $\sin\theta = \tan\theta\cdot\dfrac{1}{\sec\theta} = \dfrac{\sqrt{x^2-1}}{x}$ より

$$I = \frac{\sqrt{(x^2-1)^3}}{3x^3}$$

~~(5点)~~ (4点)