MS2 前中 2018

微積 I

2016年度試験答案用紙

番号　　氏名

1.(1) $y' = \frac{2}{x+1} - \frac{1}{x} - \frac{1}{x-1} = \frac{1-3x}{x(x-1)(x+1)}$

(2) $y' = \frac{1}{3}\cdot\frac{2x}{x^2+1} + \frac{3}{2}\cdot\frac{1}{x} = \frac{13x^2+9}{6x(x^2+1)}$

(3) $y' = \pi x^{\pi-1}$　(4) $y' = \frac{e^x}{e^x-2}$

(5) $y' = 2(\log_2 x)\cdot\frac{1}{x\log 2} = \frac{2\log_2 x}{x\log 2} = \frac{2\log x}{x(\log 2)^2}$

(6) $y' = \frac{1}{\log 3}\left(\frac{1}{x-4} + \frac{1}{2}\cdot\frac{2x}{x^2+3}\right) = \frac{2x^2-4x+3}{\cancel{2}(x-4)(x^2+3)\log 3}$

(7) $y' = \cos^{-1}x - \frac{x}{\sqrt{1-x^2}}$

(8) $y' = \frac{1}{\sqrt{1-\frac{x^2}{2}}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2-x^2}}$

(9) $y' = -2x\sin x^2$　(10) $y' = \frac{1}{1+x}\cdot\left(-\frac{1}{2\sqrt{x}}\right) = -\frac{1}{2\sqrt{x}(1+x)}$

(11) $y' = \frac{1}{4}(x^3+6x-7)^{-\frac{3}{4}}\cdot(3x^2+6) = \frac{3x^2+6}{4\cdot\sqrt[4]{(x^3+6x-7)^3}}$

(12) $y' = -\frac{2}{3}(2-x)^{-\frac{5}{3}}\cdot(-1) = \frac{2}{3(2-x)\sqrt[3]{(2-x)^2}}$

(13) $y' = -\mathrm{cosec}^2 x$　(14) $y' = \frac{\cos x}{2\sqrt{1+\sin x}}$

(15) $y' = 6\sin^2(2x+1)\cdot\cos(2x+1)$

(16) $y' = 4x^3-6x^2+1$　(17) $y' = \left(1-\frac{2}{x^2+1}\right)' = \frac{4x}{(x^2+1)^2}$

(18) $y' = (x^2+1)' = \boxed{2x}$　(19) $y' = (x^3+1)' = 3x^2$

(20) $y' = \frac{2x+1-2(x+3)}{(2x+1)^2} = \frac{-5}{(2x+1)^2}$

(21) $y' = \frac{3(x^2-5)-(3x-1)\cdot 2x}{(x^2-5)^2} = \frac{-3x^2+2x-15}{(x^2-5)^2}$

(22) $y' = -2x(2x-3)(x+2) + 2(1-x^2)(x+2) + (1-x^2)(2x-3)$
$= -8x^3-3x^2+16x+1$

ok

(23) $y' = -6x^{-4} - x^{-3} = -\frac{6+x}{x^4}$

(24) $y' = x^3 + \frac{16}{x^5}$　(25) $y' = -\frac{x^{-3}}{2} - \frac{5}{2x^4} = -\frac{x+5}{2x^4}$

(26) $y' = \frac{4}{5}x^{-\frac{1}{5}} - \frac{1}{4}x^{-\frac{5}{4}}$　(27) $y' = \frac{1}{6}x^{-\frac{5}{6}} + \frac{5}{4}x^{\frac{1}{4}}$
$= \frac{1}{6\cdot\sqrt[6]{x^5}} + \frac{5}{4}\cdot\sqrt[4]{x}$

(28) $y' = (x^{-\frac{3}{2}})' = -\frac{3}{2}x^{-\frac{5}{2}} = -\frac{3}{2x^2\sqrt{x}}$

(29) $y' = 2\sqrt{x} + \frac{2x-1}{2\sqrt{x}} = \frac{6x-1}{2\sqrt{x}}$

(30) $y' = -3\cos(2-3x)$　(31) $y' = \frac{\cos(2x-3)}{2}$

(32) $y' = 4\cos 4x\cos 5x - 5\sin 4x\sin 5x$

(別) $y' = \left\{\frac{1}{2}(\sin 9x - \sin x)\right\}' = \frac{9}{2}\cos 9x - \frac{1}{2}\cos x$

(33) $y' = (2x-1)e^x + (x^2-x+1)e^x = (x^2+x)e^x$

(34) $y' = e^x\tan x + \frac{e^x}{\cos^2 x} = \frac{e^x(\sin x\cos x+1)}{\cos^2 x}$

(35) $y' = (3e^{-2x})' = -\frac{6}{e^{2x}}$　(36) $y' = 5^x\log 5$

4(2) $\boxed{別}$ $y = \frac{1}{2}x^2\{\sin(2x+1+x-2) + \sin(2x+1-x+2)\}$

$y = \frac{1}{2}x^2\{\sin(3x-1) + \sin(x+3)\}$

$\therefore y' = x\{\sin(3x-1)+\sin(x+3)\} + \frac{1}{2}x^2\{3\cos(3x-1) + \cos(x+3)\}$

2.(1) 与式 $= \lim_{h\to 0}\frac{af(a+h) - af(a) - hf(a)}{h}$

$= \lim_{h\to 0}\left\{a\cdot\frac{f(a+h)-f(a)}{h} - f(a)\right\} = af'(a) - f(a)$ (1点)

(2) 与式 $= \lim_{h\to 0}\frac{(a^2+2ah+h^2)f(a+h) - a^2f(a)}{h}$

$= \lim_{h\to 0}\left\{\frac{a^2\{f(a+h)-f(a)\}}{h} + 2af(a+h) + hf(a+h)\right\}$

$= a^2f'(a) + 2af(a)$　$\because \lim_{h\to 0}f(a+h) = f(a)$ (3点)

(3) 与式 $= \lim_{x\to a}\frac{f(x)\cos a - f(a)\cos a + f(a)\cos a - f(a)\cos x}{x-a}$

$= \lim_{x\to a}\left\{\frac{f(x)-f(a)}{x-a}\cdot\cos a - f(a)\cdot\frac{\cos x-\cos a}{x-a}\right\}$

$= f'(a)\cos a - f(a)(\cos x)'_{x=a} = f'(a)\cos a + f(a)\sin a$ (3点)

3.(1) $f(x) = \sqrt[3]{x-1}$ とおくと $f'(x) = \frac{1}{3\cdot\sqrt[3]{(x-1)^2}}$

$\therefore f'(2) = \frac{1}{3}$　与式 $= \lim_{x\to 2}\frac{1}{\frac{f(x)-f(2)}{x-2}} = \frac{1}{f'(2)} = \boxed{3}$

(2) $t = -x$ とおくと $x\to-\infty$ のとき $t\to\infty$ だから

与式 $= \lim_{t\to\infty}\frac{-5t-2}{\sqrt{t^2-2t+3}} = \lim_{t\to\infty}\frac{-5-\frac{2}{t}}{\sqrt{1-\frac{2}{t}+\frac{3}{t^2}}} = \boxed{-5}$ (2点)

4.(1) $y' = -3e^{-3x}(\cos 3x - \sin 3x) + e^{-3x}(-3\sin 3x - 3\cos 3x)$

$= -6e^{-3x}\cos 3x$

(2) $y' = 2x\sin(2x+1)\cos(x-2) + 2x^2\cos(2x+1)\cos(x-2)$
$- x^2\sin(2x+1)\sin(x-2)$

(3) $y' = \{\log|3x+1| - \log(x^2+1)\}' = \frac{3}{3x+1} - \frac{2x}{x^2+1}$

$= \frac{3-2x-3x^2}{(3x+1)(x^2+1)}$

(4) $y' = \frac{1}{1+x^2} - \frac{1}{(\tan^{-1}\frac{1}{x})^2}\left(\tan^{-1}\frac{1}{x}\right)'$

$= \frac{1}{1+x^2} - \frac{1}{(\tan^{-1}\frac{1}{x})^2}\cdot\frac{1}{1+\frac{1}{x^2}}\cdot\left(-\frac{1}{x^2}\right)$

$= \frac{1}{1+x^2} + \frac{1}{(1+x^2)(\tan^{-1}\frac{1}{x})^2} = \frac{(\tan^{-1}\frac{1}{x})^2+1}{(1+x^2)(\tan^{-1}\frac{1}{x})^2}$

(4点)

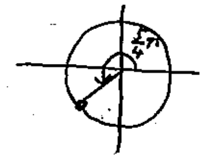
5. $0 \leqq y \leqq \boxed{\pi}$ (1) ($\cos^{-1}$の値域), $y = \cos^{-1}x \Leftrightarrow \cos y = \boxed{x}$ (2)

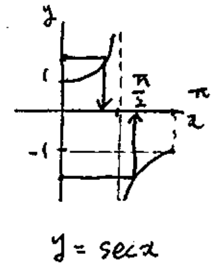
$\therefore \cos(\cos^{-1}x) = \cos y = \boxed{x}$ (3)

一方 $\cos^{-1}(\cos x) = x$ は正しくない. 反例として

$\cos^{-1}\left(\cos\frac{5}{4}\pi\right) = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (4)

$= \pi - \cos^{-1}\frac{1}{\sqrt{2}}$ (5) $= \frac{3}{4}\pi$ (6)





$y = \sec x$ を $0 \leqq x < \frac{\pi}{2}, \frac{\pi}{2} < x \leqq \pi$,

$|y|$ (7) $\geqq 1$ で考えれば 逆関数が存在し

それを $y = \sec^{-1}x$ と表す

定義域 $|x| \geqq 1$ (8), 値域 $0 \leqq y < \frac{\pi}{2}, \frac{\pi}{2} < y \leqq \pi$

$\sec^{-1}\sqrt{2} = \frac{\pi}{4}$ (9), $\sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$ (10), $\sec^{-1}(-2) = \frac{2\pi}{3}$ (11)

$(\sec^{-1}x)'$ を求める. 逆関数の微分法から $x = \sec y$ (12) とすれば

$(\sec^{-1}x)' = \frac{1}{(\sec y)'} = \frac{1}{\sec y \cdot \tan y}$ (13), $\tan y = \pm\sqrt{\sec^2 y - 1}$

$\tan y = \begin{cases} \sqrt{x^2-1} & (x \geqq 1) \\ -\sqrt{x^2-1} & (x \leqq -1) \end{cases}$ (14) (15)

→ (14) $\tan(\sec^{-1}x)$ は だめ. これは場合分けされているから.

結局 この2つをまとめると $(\sec^{-1}x)' = \frac{1}{|x|\sqrt{x^2-1}}$ (16) となる

(10点)

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6. $y = \sinh^{-1}x \Leftrightarrow x = \sinh y$ となる

→ 実は $\sinh^{-1}x = \log(x + \sqrt{1+x^2})$ (授業で学習した)

$(\sinh^{-1}x)' = \frac{dy}{dx} = \frac{1}{(\sinh y)'} = \frac{1}{\cosh y}$

ところで $\cosh^2 y - \sinh^2 y = 1$, $\cosh y > 0$ となる

$\cosh y = \sqrt{1 + \sinh^2 y} = \sqrt{1+x^2}$

$\therefore (\sinh^{-1}x)' = \frac{1}{\sqrt{1+x^2}}$

(5点)