ES4 前中、2017

2016年度試験答案用紙

番号　　氏名

1.(1) [1] $F(s) = -\frac{d}{ds}\mathcal{L}[\sinh 3t] = -\left(\frac{3}{s^2-9}\right)' = \frac{6s}{(s^2-9)^2}$

[2] [1] から $F(s) = \frac{6(s-2)}{(s-5)^2(s+1)^2} = \frac{6(s-2)}{(s^2-4s-5)^2} = \frac{6s}{(s-3)^2(s+3)^2}$

[3] [1] から $F(s) = \frac{6s\,e^{-s}}{(s^2-9)^2}$

[4] [1] から $F(s) = \frac{6s^2}{(s^2-9)^2} - 0 = \frac{6s^2}{(s^2-9)^2}$

[5] [1] から $F(s) = \frac{6}{(s^2-9)^2}$

[6] $\lim_{t\to+0}\frac{\sinh 3t}{t} = 3$ だから $\mathcal{L}[\sinh 3t] = \frac{3}{s^2-9}$ とあわせて

$F(s) = \int_s^\infty \frac{3}{\sigma^2-9}\,d\sigma = \frac{1}{2}\left[\log\left|\frac{\sigma-3}{\sigma+3}\right|\right]_s^\infty = \frac{1}{2}\log\left|\frac{s+3}{s-3}\right|$

[7] $F(s) = \mathcal{L}[t]\mathcal{L}[\sinh 3t] = \frac{1}{s^2}\frac{3}{s^2-9} = \frac{3}{s^2(s^2-9)}$

[8] $sF - 1 + F = \frac{1}{s^2+1}$ より $(s+1)F = 1 + \frac{1}{s^2+1} = \frac{s^2+2}{s^2+1}$

$\therefore F(s) = \frac{s^2+2}{(s+1)(s^2+1)}$

(2) [1] 与式 $= \frac{(s-3)+1}{(s-3)^2+4} = \frac{s-3}{(s-3)^2+4} + \frac{1}{2}\frac{2}{(s-3)^2+4}$

$\therefore f(t) = e^{3t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)$

[2] 与式 $= \frac{1}{(s-3)(s-2)} = \frac{1}{s-3} - \frac{1}{s-2}$ $\therefore f(t) = e^{3t} - e^{2t}$

(別) 与式 $= \frac{1}{(s-\frac{5}{2})^2 - \frac{1}{4}} = 2\frac{\frac{1}{2}}{(s-\frac{5}{2})^2-\frac{1}{4}}$ $\therefore f(t) = 2e^{\frac{5}{2}t}\sinh\frac{t}{2}$

[3] $\mathcal{L}^{-1}\left[\frac{5}{s^4}\right] = \frac{5}{3!}t^3 = \frac{5}{6}t^3$ より $F(s) = \frac{5}{6}t^3e^{-3t}$

[4] 与式 $= \frac{1}{s+2} + \frac{-2}{s-2} + \frac{4}{s-3}$ より

$f(t) = e^{-2t} - 2e^{2t} + 4e^{3t}$

2.(1) $s^2X + sX - 2X = (s^2+s-2)X$ [1]: $s^2+s-2 = (s+2)(s-1)$

$\mathcal{L}[\delta(t)] = 1$ $\therefore X = \frac{1}{s^2+s-2}$ [2]: 1 (3) $\frac{1}{s^2+s-2}$

$X = \frac{1}{3}\left(\frac{1}{s-1} - \frac{1}{s+2}\right)$ より $x(t) =$ [4]: $\frac{1}{3}(e^t - e^{-2t})$

2(2) 3(例) [1] $\mathcal{L}[e^t * \delta(t)] = \mathcal{L}[e^t]\mathcal{L}[\delta(t)] = \frac{1}{s-1}$

(例) [2] $\mathcal{L}[t * \sin 3t] = \mathcal{L}[t]\mathcal{L}[\sin 3t] = \frac{3}{s^2(s^2+9)}$

2(3) [1] $\mathcal{L}^{-1}\left[\frac{1}{s^2+4s+13}\right] = \mathcal{L}^{-1}\left[\frac{1}{(s+2)^2+9}\right] = \frac{1}{3}e^{-2t}\sin 3t$ より

$\mathcal{L}^{-1}\left[\frac{F(s)}{s^2+4s+13}\right] = \frac{1}{3}\int_0^t f(t-\tau)e^{-2\tau}\sin 3\tau\,d\tau$

[2] $\mathcal{L}^{-1}\left[\frac{1}{s^2-6s+9}\right] = \mathcal{L}^{-1}\left[\frac{1}{(s-3)^2}\right] = e^{3t}\mathcal{L}\left[\frac{1}{s^2}\right] = te^{3t}$ より

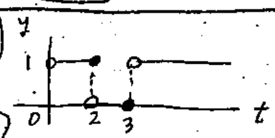
$\mathcal{L}^{-1}\left[\frac{F(s)}{s^2-6s+9}\right] = \int_0^t f_2(t-\tau)\,\tau e^{3\tau}\,d\tau$

2(4) 左辺: $X\mathcal{L}[e^t] = \frac{1}{s-1}X$ [1]: $\frac{1}{s-1}$ [2] $\frac{1}{s^2}$

右辺: $\frac{1}{s^2}$ $\therefore X = \frac{s-1}{s^2}$ [3] $\frac{s-1}{s^2} = \frac{1}{s} - \frac{1}{s^2}$

$\therefore x(t) = 1 - t$ [4] $1-t$

(5) [1] $f(t) = U(t) - U(t-2) + U(t-3)$



[2] $\mathcal{L}[f(t)] = \frac{1}{s} - \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s}$ $U(t)$ は 1でもよい

$= \frac{1 - e^{-2s} + e^{-3s}}{s}$

(6) $\mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right] = \cos 2t$ (例), $\mathcal{L}^{-1}\left[\frac{1}{s^2+4}\right] = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{2}{s^2+4}\right] = \frac{1}{2}\sin 2t$ [1](例)

$\therefore \mathcal{L}^{-1}[F(s)] = \cos 2t * \frac{1}{2}\sin 2t = \int_0^t \cos 2\tau\left(\frac{1}{2}\sin 2(t-\tau)\right)d\tau$ [2](例)

$= \frac{1}{4}\int_0^t \left(\sin 2t + \sin(\cancel{4\tau - 2t})\right)$ [3](例) [4](例)

$2t - 4\tau$

$= \frac{1}{4}\left[\tau\sin 2t + \frac{1}{4}\cos(4\tau - 2t)\right]_0^t$

$= \frac{1}{4}(t\sin 2t + 0) = \frac{1}{4}t\sin 2t$ [5]

[1] $\frac{1}{2}\sin 2t$ [2] $\frac{1}{2}\sin 2(t-\tau)$ [3] $\frac{1}{4}$ [4] $\sin(4\tau - 2t)$

[5] $\frac{1}{4}t\sin 2t$

3.(1) $F(s) = \int_0^2 te^{-st}dt + \int_2^4 (4-t)e^{-st}dt$

$= \left[-\frac{t}{s}e^{-st}\right]_0^2 + \frac{1}{s}\int_0^2 e^{-st}dt + \left[-\frac{1}{s}(4-t)e^{-st}\right]_2^4 - \frac{1}{s}\int_2^4 e^{-st}dt$

$= -\frac{2}{s}e^{-2s} + \left[-\frac{1}{s^2}e^{-st}\right]_0^2 + \frac{2}{s}e^{-2s} + \left[\frac{1}{s^2}e^{-st}\right]_2^4$

$= \frac{1}{s^2}(1 - e^{-2s}) + \frac{1}{s^2}(e^{-4s} - e^{-2s}) = \frac{1}{s^2}(e^{-2s} - 1)^2$ 4

(2) $f(t) = tU(t) - 2(t-2)\,U(t-2) + ((t-4))U(t-4)$ [1] [2]

だから $\mathcal{L}[tU(t)] = \frac{1}{s^2}$ [3], $\mathcal{L}[2(t-2)U(t-2)] = \frac{2}{s^2}e^{-2s}$ [4]

$\mathcal{L}[(t-4)U(t-4)] = \frac{1}{s^2}e^{-4s}$ [5] $\therefore F(s) = \frac{1}{s^2}(1 - e^{-2s})^2$ [6] (例)

(3) $g(t) - g(t-4)U(t-4) = f(t)$ だから

$G(s) - e^{-4s}G(s) = F(s)$ 3点

$\therefore G(s) = \frac{1}{1 - e^{-4s}}\cdot\frac{1}{s^2}(1 - e^{-2s})^2 = \frac{(1 - e^{-2s})^2}{s^2(1 + e^{-2s})(1 - e^{-2s})}$

$= \frac{1 - e^{-2s}}{s^2(1 + e^{-2s})} = \frac{e^{2s} - 1}{s^2(e^{2s} + 1)}$ 4

4. $\alpha > -1$ のとき $\mathcal{L}[t^\alpha]$ を求めると

$\mathcal{L}[t^\alpha] = \int_0^\infty e^{-st}t^\alpha dt \underset{x = st,\ s>0}{=} \int_0^\infty e^{-x}\left(\frac{x}{s}\right)^\alpha\frac{1}{s}\,dx$ (1)

$= \frac{1}{s^{\alpha+1}}\int_0^\infty e^{-x}x^{(\alpha+1)-1}dx = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$

$\Gamma(p) = \int_0^\infty e^{-x}x^{p-1}dx$

$= \left[-e^{-x}x^{p-1}\right]_0^\infty + (p-1)\int_0^\infty e^{-x}x^{p-2}dx = (p-1)\Gamma(p-1)$

(2) $\alpha+1$
(3) α
(4) $\alpha+1$
(5) $p-1$

$\mathcal{L}[t\sqrt{t}] = \mathcal{L}[t^{\frac{3}{2}}] = \frac{\Gamma(\frac{5}{2})}{s^{\frac{5}{2}}} = \frac{1}{s^{\frac{5}{2}}}\frac{3}{2}\Gamma\left(\frac{3}{2}\right)$ (6) $\frac{5}{2}$ (7) $\frac{5}{2}$

$= \frac{1}{s^{\frac{5}{2}}}\left(\frac{3}{2}\cdot\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)$ …(*) (8) $\frac{3}{4}$

$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty \frac{e^{-x}}{\sqrt{x}}dx = \int_0^\infty 2e^{-u^2}du = \sqrt{\pi}$ (9) $\frac{3}{4}\sqrt{\pi}$

$u = \sqrt{x} \to du = \frac{1}{2\sqrt{x}}dx$

$\therefore (*) = \frac{3}{4}\sqrt{\pi}\cdot\frac{1}{s^{\frac{5}{2}}}$

5 (1) $\mathcal{L}[t x(t)] = \boxed{-\dfrac{dX}{ds}}$

(2) $\mathcal{L}[x''] = s^2 X - 1$ より

$$\mathcal{L}[t x''] = -\frac{d}{ds}\left(s^2 X - 1\right) = \boxed{-s^2\frac{dX}{ds} - 2sX}$$

(3) $$-s^2\frac{dX}{ds} - 2sX + 2sX + \frac{dX}{ds} + 2X = \frac{2}{s-1}$$

$$\therefore (s^2-1)\frac{dX}{ds} - 2X = -\frac{2}{s-1}$$

$$\frac{dX}{ds} - \boxed{\frac{2}{s^2-1}}X = \boxed{-\frac{2}{(s-1)^2(s+1)}}$$

[1] $\dfrac{2}{s^2-1}$ [2] $-\dfrac{2}{(s-1)^2(s+1)}$

(4) $\dfrac{dX}{ds} - \dfrac{2}{s^2-1}X = 0$ を解くと、$\dfrac{1}{X}\dfrac{dX}{ds} = \dfrac{2}{s^2-1}$

$$\log|X| = \int \frac{2}{s^2-1}ds = \log\left|\frac{s-1}{s+1}\right| + C \quad \therefore X = \frac{C(s-1)}{s+1}$$

C を s の関数 $u = u(s)$ に置き換えて②に代入すると

$$\frac{s-1}{s+1}\frac{du}{ds} = -\frac{2}{(s-1)^2(s+1)} \rightarrow \frac{du}{ds} = -\frac{2}{(s-1)^3}$$

$$\therefore u = \frac{1}{(s-1)^2} \quad \left(\lim_{t\to+0} x(0) = 0 \text{ より } C = 0\right)$$

より $$X = \frac{1}{(s-1)^2}\cdot\frac{s-1}{s+1} = \frac{1}{s^2-1} \quad \therefore x(t) = \sinh t$$ 6

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6. $$\mathcal{L}\left[\int_0^\infty \frac{\cos tx}{x^2+a^2}dx\right] = \int_0^\infty \frac{\mathcal{L}[\cos tx]}{x^2+a^2}dx$$

$$= \int_0^\infty \frac{1}{x^2+a^2}\cdot\overset{(1)}{\boxed{\frac{s}{s^2+x^2}}}dx = \overset{(2)}{\boxed{\frac{s}{s^2-a^2}}}\int_0^\infty\left(\frac{1}{x^2+a^2} - \overset{(3)}{\boxed{\frac{1}{x^2+s^2}}}\right)dx$$

$$= \frac{s}{s^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a} - \frac{1}{s}\tan^{-1}\frac{x}{s}\right]_0^\infty$$

$$= \frac{s}{s^2-a^2}\cdot\frac{\pi}{2}\left(\frac{1}{a} - \frac{1}{s}\right) = \overset{(4)}{\boxed{\frac{\pi}{2a}\ \frac{1}{s+a}}}$$

$$\therefore \int_0^\infty \frac{\cos tx}{x^2+a^2}dx = \frac{\pi}{2a}\mathcal{L}^{-1}\left[\frac{1}{s+a}\right] = \overset{(5)}{\boxed{\frac{\pi}{2a}e^{-at}}}$$ 5

- 3(1)を(2)の方法で解いた答案には点を与えない。
定義に従ってそのまま計算する問題 (2)を見れば何を要求しているか分かるはず
- 5(2) $-\dfrac{d}{ds}(s^2X-1)$ は点を与えない。計算せよ。
- 5(4) 1階線形微分方程式ぐらい解けなくてはだめ。