DC3 学末 2017

2016年度試験答案用紙

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1. $p = g(p) + \boxed{\{x g'(p) + f'(p)\}\dfrac{dp}{dx}}^{(1)}$ $\therefore \dfrac{dp}{dx} = \boxed{\dfrac{p-g(p)}{xg'(p)+f'(p)}}^{(2)}$

$\dfrac{dp}{dx} = \dfrac{1}{\boxed{\dfrac{dx}{dp}}^{(3)}}$ $\therefore \dfrac{1}{\frac{dx}{dp}} = \dfrac{p-g(p)}{xg'(p)+f'(p)}$

$\dfrac{dx}{dp} = \dfrac{g'(p)}{p-g(p)}x + \dfrac{f'(p)}{p-g(p)}$ $\therefore \dfrac{dx}{dp} + \boxed{\dfrac{g'(p)}{g(p)-p}}^{(4)} x = \boxed{\dfrac{f'(p)}{p-g(p)}}^{(5)}$

$\dfrac{dx}{dp} + \dfrac{g'(p)}{g(p)-p}x = 0$ を解く → $\log|x| = \int \dfrac{g'(p)}{p-g(p)}dp + C$

$\therefore x = C\exp\left(\int \boxed{\dfrac{g'(p)}{p-g(p)}}^{(6)} dp\right)$

$g(p_0) = p_0$ なる実数の定数があれば $y = \boxed{p_0 x + f(p_0)}^{(7)}$

∵ 上式より $y' = p_0$ つまり $p = p_0$ より ①の右辺 $= xg(p_0) + f(p_0)$

$= p_0 x + f(p_0)$, これは y に等しい

$\boxed{y = x(2p-4p^2) + p^4 - \dfrac{p^3}{3}}$ $(*)$ なので $g = 2p - 4p^2,\ f = p^4 - \dfrac{p^3}{3}$

$\therefore \dfrac{g'(p)}{g(p)-p} = \boxed{\dfrac{2}{p}}^{(8)}$, $\dfrac{f'(p)}{p-g(p)} = \boxed{p}^{(9)}$

$\therefore \dfrac{dx}{dp} + \dfrac{2}{p}x = p$. $\dfrac{dx}{dp} + \dfrac{2}{p}x = 0$ を解いて

$x = \boxed{\dfrac{C}{p^2}}^{(10)}$ を, C を $u = u(p)$ に置きかえて $\dfrac{u'}{p^2} = p$

$\therefore u = \dfrac{1}{4}p^4 + C$ $\therefore x = \boxed{\dfrac{p^2}{4}}^{(11)} + \dfrac{C}{p^2}$, これを $(*)$ に代入

$y = \left(\dfrac{p^2}{4} + \dfrac{C}{p^2}\right)(2p-4p^2) + p^4 - \dfrac{p^3}{3} = \boxed{\dfrac{1}{6}}^{(12)} p^3 + 2C\left(\boxed{\dfrac{1}{p} - 2}^{(13)}\right)$

$g(p_0) = p_0$ となる実定数は $2p_0 - 4p_0^2 = p_0$ から $y = 0$ と

$y = \dfrac{1}{4}x + f\left(\dfrac{1}{4}\right) = \boxed{\dfrac{1}{4}x - \dfrac{1}{768}}^{(14)} = \boxed{\dfrac{1}{4}\left(x - \dfrac{1}{192}\right)}$

11段

2. $\boxed{\lambda^4 - \lambda^2 - 6 = 0}^{(1)}$ $(\lambda^2-3)(\lambda^2+2) = 0$

$\therefore \lambda = \boxed{\pm\sqrt{3}}^{(2)}$ $\lambda = \boxed{\pm\sqrt{2}\,i}^{(3)}$

$\therefore y = \boxed{C_1 e^{\sqrt{3}x} + C_2 e^{-\sqrt{3}x} + C_3\cos\sqrt{2}x + C_4\sin\sqrt{2}x}^{(4)}$

3. $A = \boxed{\begin{pmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ 2 & -4 & 0 \end{pmatrix}}^{(1)}$, $|A - \lambda E| = \begin{vmatrix} 1-\lambda & 2 & 1 \\ -1 & 4-\lambda & 1 \\ 2 & -4 & -\lambda \end{vmatrix} = \boxed{(1-\lambda)}^{(2)}\boxed{(2-\lambda)}^{(3)2}$

$\boxed{\lambda - 2\text{でもOK}}$

$\lambda_1 = 1,\ \lambda_2 = 2$

$\lambda_1 = 1$: $\begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & 1 \\ 2 & -4 & -1 \end{pmatrix} \to \begin{pmatrix} -1 & 3 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $\therefore x_1 = \begin{pmatrix} 1 \\ 1 \\ \boxed{-2} \end{pmatrix}^{(4)}$

$\lambda_2 = 2$: $\begin{pmatrix} -1 & 2 & 1 \\ -1 & 2 & 1 \\ 2 & -4 & -2 \end{pmatrix} \to \begin{pmatrix} -1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\therefore x_2 = \begin{pmatrix} \boxed{2} \\ 1 \\ 0 \end{pmatrix}^{(5)}$,

$x_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $x = Py$ と変換して

$P\dfrac{dy}{dt} = APy$ $\therefore \dfrac{dy}{dt} = P^{-1}APy = \boxed{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}}^{(6)} y$

$y = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ とおくと $\begin{pmatrix} \frac{dX}{dt} \\ \frac{dY}{dt} \\ \frac{dZ}{dt} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$

$\therefore X = C_1\boxed{e^t}^{(7)}$ $Y = C_2\boxed{e^{2t}}^{(8)}$ $Z = C_3 e^{2t}$

$\therefore x = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} C_1 e^t \\ C_2 e^{2t} \\ C_3 e^{2t} \end{pmatrix} = \begin{pmatrix} \boxed{C_1 e^t + 2C_2 e^{2t} + C_3 e^{2t}}^{(9)} \\ \boxed{C_1 e^t + C_2 e^{2t}}^{(10)} \\ \boxed{-2C_1 e^t + C_3 e^{2t}}^{(11)} \end{pmatrix}$

8段

4. (1) 特性方程式 $\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$, $(\lambda+2)(\lambda-1)(\lambda-3) = 0$

$\therefore \lambda = -2, 1, 3$ [1] $y_1 = \boxed{e^{-2x}}$ [2] $y_2 = \boxed{e^x}$ [3] $y_3 = \boxed{e^{3x}}$

(2) $W(e^{-2x}, e^x, e^{3x}) = \begin{vmatrix} e^{-2x} & e^x & e^{3x} \\ -2e^{-2x} & e^x & 3e^{3x} \\ 4e^{-2x} & e^x & 9e^{3x} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -2 & 1 & 3 \\ 4 & 1 & 9 \end{vmatrix} e^{2x}$

$= \boxed{30e^{2x}}$

(3) $\eta = \dfrac{1}{(D+2)(D-1)(D-3)} e^{-2x} = \dfrac{1}{D+2}\left(\dfrac{1}{D^2-4D+3} e^{-2x}\right)$

$= \dfrac{1}{D+2}\left(\dfrac{1}{4+8+3} e^{-2x}\right) = \dfrac{1}{15}\dfrac{1}{D+2} e^{-2x}$

$= \dfrac{1}{15} e^{-2x} \dfrac{1}{D-2+2} 1 = \dfrac{1}{15} e^{-2x} \dfrac{1}{D} 1$

$= \dfrac{1}{15} e^{-2x} \int dx = \dfrac{x}{15} e^{-2x}$

(4) $\boxed{y = C_1 e^{-2x} + C_2 e^x + C_3 e^{3x} + \dfrac{x}{15} e^{-2x}}$

5. $y' + \boxed{\dfrac{1-6x^3}{2x}}^{(1)} y = \boxed{-\dfrac{1}{2}x^2\sqrt{x}}^{(2)} y^2$; (1) $\left(\dfrac{1}{2x} - 3x^2\right)$ も正解

(　　)ならば ×

$y' + \dfrac{1-6x^3}{2x} y = 0$ から $\log|y| = \int \dfrac{6x^3-1}{2x}dx + C = \boxed{x^3 - \dfrac{1}{2}\log x}^{(3)} + C$

$\therefore y = C\boxed{\dfrac{e^{x^3}}{\sqrt{x}}}^{(4)}$, C を $u = u(x)$ に置き換え ②に代入して計算

$u' \dfrac{e^{x^3}}{\sqrt{x}} = -\dfrac{1}{2}x^2\sqrt{x}\,\dfrac{e^{2x^3}}{x} u^2$ $\therefore \dfrac{1}{u^2}u' = \boxed{-\dfrac{1}{2}x^2 e^{x^3}}^{(5)}$

$-\dfrac{1}{u} = -\dfrac{1}{2}\int x^2 e^{x^3} dx + C = -\dfrac{1}{6}e^{x^3} + C$ $\therefore u = \dfrac{1}{\boxed{\frac{1}{6}e^{x^3}}^{(6)} + C}$

以上より ①の一般解は $y = \boxed{\dfrac{e^{x^3}}{\left(\frac{1}{6}e^{x^3} + C\right)\sqrt{x}}}^{(7)}$

; (7) $\dfrac{6e^{x^3}}{(e^{x^3} + C)\sqrt{x}}$ も正解

6. (1) $y^{(3)} + 3y'' - y' - 3y = 0$ の特性方程式は $\lambda^3 + 3\lambda^2 - \lambda - 3 = 0$

$(\lambda+3)(\lambda+1)(\lambda-1) = 0$ より上の基本解は e^{-3x}, e^{-x}, e^x.

1つの解をもとめる. $-3 - D + 3D^2 + D^3 \overline{)\,3x^2 + 20x}$ 　$-x^2 - 6x$

$3x^2 + 2x - 6$

以上より一般解は　$18x + 6$

$y = C_1 e^{-3x} + C_2 e^{-x} + C_3 e^x - x^2 - 6x$　$18x + 6$

0

⑤

(2) $y_1 = e^{-3x},\ y_2 = e^{-x},\ y_3 = e^x$

$W(0) = W(y_1, y_2, y_3)(0) = \begin{vmatrix} 1 & 1 & 1 \\ -3 & -1 & 1 \\ 9 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ -3 & -1 & 2 \\ 9 & 1 & 0 \end{vmatrix} = -2\begin{vmatrix} 1 & 1 \\ 9 & 1 \end{vmatrix}$

$= \boxed{16}^{(1)}$

$\therefore W(y_1, y_2, y_3) = 16\exp\left(-\int_0^x \boxed{3}^{(2)} d\xi\right) = \boxed{16e^{-3x}}^{(3)}$

(3) (2) と (1) から $\begin{pmatrix} 1 & 1 & 1 \\ -3 & -1 & 1 \\ 9 & 1 & 1 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \\ 6 \end{pmatrix}$ を解けばよい

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -3 & -1 & 1 & 9 \\ 9 & 1 & 1 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 4 & 9 \\ 0 & -8 & -8 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 4 & 9 \\ 0 & 0 & 8 & 42 \end{array}\right)$

$C_3 = \dfrac{21}{4}$, $2C_2 = 9 - 21 = -12$ $\therefore C_2 = -6$, $C_1 = 6 - \dfrac{21}{4} = \dfrac{3}{4}$

⑤

$\therefore y = \dfrac{3}{4}e^{-3x} - 6e^{-x} + \dfrac{21}{4}e^x - x^2 - 6x$

両方正解の場合1点とする

7 (1) [1] $P_y = \boxed{e^x \cos y}$ [2] $Q_x = \boxed{e^x \cos y}$

(2) (1)から①は完全微分方程式なので $u=u(x,y)$ で $u_x = P$, $u_y = Q$ となるものを求める.

ここがCなら ×

$u_x = 1 + e^x \sin y$ から $u = x + e^x \sin y + \boxed{\varphi(y)}$ …… (i)

$\therefore u_y = e^x \cos y + \varphi'(y) = Q = e^x \cos y + 1$ $\therefore \varphi(y) = y$

これと(i)から一般解は $x + y + e^x \sin y = C$ (5行)

8 (1) ③は $y'' - \dfrac{x}{x-1}y' + \dfrac{1}{x-1}y = 0$ なので $P = -\dfrac{x}{x-1}$, $Q = \dfrac{1}{x-1}$ と表せば $P + xQ = 0$ ∴ 1つの解 $\boxed{y_1 = x}$

(2) $y = xu$ より $y' = xu' + u$, $y'' = xu'' + 2u'$ だから①に代入して

$(x-1)(xu''+2u') - x^2u' = (x-1)^2$

$x(x-1)u'' + (2x - x^2 - 2)u' = (x-1)^2$ $\therefore S(x) = \dfrac{-x^2+2x-2}{x(x-1)}$,

$T(x) = \dfrac{x-1}{x}$ (4行)

(3) [1] $w' + \dfrac{-x^2+2x-2}{x(x-1)}w = 0 \to \log|w| = \displaystyle\int \frac{x^2-2x+2}{x(x-1)}dx + C$

$\log|w| = \displaystyle\int \left(1 + \frac{1}{x-1} - \frac{2}{x}\right)dx + C = x + \log\frac{x-1}{x^2} + C$

$\therefore w = \dfrac{C(x-1)e^x}{x^2}$ (3行)

[2] $v = v(x)$, $w = v \cdot \dfrac{(x-1)e^x}{x^2}$ として④に代入すれば

$v'\dfrac{(x-1)e^x}{x^2} = \dfrac{x-1}{x}$ $\therefore v' = xe^{-x}$

$v = -xe^{-x} - e^{-x} + C_1$ より

$w = \left\{-(x+1)e^{-x} + C_1\right\}\dfrac{(x-1)e^x}{x^2} = \dfrac{C_1(x-1)e^x}{x^2} - 1 + \dfrac{1}{x^2}$ (4行)

(4) $u = C_1\displaystyle\int\left(\frac{1}{x} - \frac{1}{x^2}\right)e^x dx - x - \frac{1}{x} + C_2$

ここで $\displaystyle\int \frac{1}{x}e^x dx = \frac{1}{x}e^x + \int \frac{1}{x^2}e^x dx$ であるから

$u = \dfrac{C_1}{x}e^x - x - \dfrac{1}{x} + C_2$ $\therefore y = C_1e^x + C_2x - x^2 - 1$ (3行)

<参考> 問題8について

(1)で $1 + P + Q = 1 - \dfrac{x}{x-1} + \dfrac{1}{x-1} = 0$ でもあるから $y_1 = e^x$ も正解である. $y_1 = e^x$ としたとき (2)〜(4)は次のようになる.

(2) $y = ue^x$ より $y' = u'e^x + ue^x$, $y'' = u''e^x + 2u'e^x + ue^x$ だから

$(x-1)(u''+2u')e^x - xu'e^x = (x-1)^2$

$u'' + 2u' - \dfrac{x}{x-1}u' = (x-1)e^{-x}$ $\therefore S(x) = \dfrac{x-2}{x-1}$, $T(x) = (x-1)e^{-x}$

(3) [1] $w' + \left(1 - \dfrac{1}{x-1}\right)w = 0$

$\log|w| = \displaystyle\int\left(\frac{1}{x-1} - 1\right)dx + C = \log(x-1) - x + C$

$\therefore w = C(x-1)e^{-x}$

[2] $w = v(x-1)e^{-x}$ とすれば これを④に代入して

$v'(x-1)e^{-x} = (x-1)e^{-x}$ $\therefore v' = 1 \to v = x + C_1$

$\therefore w = (x+C_1)(x-1)e^{-x} = C_1(x-1)e^{-x} + x(x-1)e^{-x}$

(4) $\displaystyle\int x(x-1)e^{-x}dx = \int x^2e^{-x}dx - \int xe^{-x}dx$

$= -x^2e^{-x} + \displaystyle\int xe^{-x}dx = -x^2e^{-x} - xe^{-x} - e^{-x}$

$C_1\displaystyle\int(x-1)e^{-x}dx = C_1\left(\int xe^{-x}dx + e^{-x}\right) = -C_1xe^{-x}$

$\therefore u = -(-x^2 - x - 1 - C_1x)e^{-x} + C_2$

$u = (-x^2 - 1 + C_1x)e^{-x} + C_2$ $[-C_1 + 1 \to C_1]$

$\therefore y = C_1x + C_2e^x - x^2 - 1$ △

<採点について>

1. (4) $-\dfrac{g'(p)}{p-g(p)}$ は1点, $\dfrac{-g'(p)}{p-g(p)}$ は正解

前者は数学の式として△ $\left(-\dfrac{g'(p)}{p-g(p)}\right)$ なら正解.

5 (1) そこでも書いてあるが $\dfrac{1}{2x} - 3x^2$ は × そのまま入れると $-3x^2$ だけに y が掛かる.