DC3 後中 2017

1. (2)

2016年度試験答案用紙

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$u = \frac{y}{x}$ とおけば $y = xu$ より $y' = u + xu'$ となるので

$u + xu' = 3u - 1$ 整理して $\frac{1}{2u-1}u' = \frac{1}{x}$

$\frac{1}{2}\log|2u-1| = \log x + C$ ∴ $2u - 1 = Cx^2$

(3) uを戻して $2\frac{y}{x} = Cx^2 + 1$ ∴ $y = Cx^3 + \frac{x}{2}$ (C:任意定数) 4行

$y(1) = 2$ より $y(1) = C + \frac{1}{2} = 2$ ∴ $C = \frac{3}{2}$

よって $\boxed{y = \frac{3}{2}x^3 + \frac{1}{2}x}$

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2. (1) $p = y'$ とおけば $p' = (p+1)^2$ より $\frac{1}{(p+1)^2}p' = 1$

$\int \frac{dp}{(p+1)^2} = \int dx + C$ ∴ $-\frac{1}{p+1} = x + C_1$

$p = \frac{1}{C_1 - x} - 1$ ∴ $y = \int\left(\frac{1}{C_1 - x} - 1\right)dx + C_2$

$y = -\log|C_1 - x| - x + C_2$ $(C_1, C_2$: 任意定数$)$ 4行

☆ $y = -\log|x + C_1| - x + C_2$ でも可.

(2) $y'(0) = p(0) = \frac{1}{C_1} - 1 = 0$ ∴ $C_1 = 1$ より

$y = -\log|1 - x| - x + C_2$

$y(0) = -\log 1 + C_2 = C_2 = 1$ ∴ $C_2 = 1$

以上より $y = -\log|1 - x| - x + 1$ 4行

☆ $y = -\log|x - 1| - x + 1$ でも可

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1. (1) $\frac{y'}{y} = \frac{3}{x}$ から $\int\frac{dy}{y} = \int\frac{3}{x}dx + C$, $\log|y| = 3\log x + C$

∴ $y = C\boxed{x^3}$[1] $y = u(x)x^3$ を①に代入

$u'x^3 + 3ux^2 - 3ux^2 = -1$, $u' = \boxed{-\frac{1}{x^3}}$[2]

$u = \boxed{\frac{1}{2x^2}}$[3] $+ C$ ∴ $y = ux^3 = \boxed{\frac{x}{2} + Cx^3}$[4] (C:任意定数)

となる.

[1] x^3 [2] $-\frac{1}{x^3}$ [3] $\frac{1}{2x^2}$

[4] $\frac{x}{2} + Cx^3$

3

(1) 特性方程式は $\lambda^2 + 2\lambda + 1 = 0$ より 解は $\lambda = -1$ (2重解)

従って一般解は $\boxed{y = (C_1 + C_2x)e^{-x}}$ $(C_1, C_2$: 任意定数$)$

(2) $\eta' = 2Ax + B$, $\eta'' = 2A$ だから これらを①に代入して $(y = \eta)$

$2A + 2(2Ax + B) + Ax^2 + Bx + C = 2x^2 - 6x - 7$

$Ax^2 + (4A + B)x + 2A + 2B + C = 2x^2 - 6x - 7$

∴ $A = \boxed{2}$, $4A + B = -6$ より $B = \boxed{-14}$, $C = -7 - 2A - 2B = \boxed{17}$

(3) $\boxed{y = (C_1 + C_2x)e^{-x} + 2x^2 - 14x + 17}$ $(C_1, C_2$: 任意定数$)$

(4) $y(0) = C_1 + 17 = 0$ ∴ $C_1 = -17$

$y' = (C_2 - C_1 - C_2x)e^{-x} + 4x - 14$ より $y'(0) = C_2 - C_1 - 14 = 0$

∴ $C_2 = C_1 + 14 = -3$ 以上により

$y = -(17 + 3x)e^{-x} + 2x^2 - 14x + 17$ 4行

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4. ①から $y = x' - 4x - t$ …③ 微分して $y' = x'' - 4x' - 1$ …④

③,④と②から $x'' - 4x' - 1 = -2x + x' - 4x - t - 5t + 1$

∴ $x'' - \boxed{5}$[1]$x' + \boxed{6}$[2]$x = \boxed{-6t + 2}$[3] …⑤

$x'' - \boxed{5}x' + \boxed{6}x = 0$ を解いて $x = C_1e^{\boxed{2}t}$[4] $+ C_2e^{\boxed{3}t}$[5] ([4] < [5])

⑤の1つの解の形を $x = At + B$ と予想すれば $x = \boxed{-t - \frac{1}{2}}$[6]

よって⑤の一般解は $x = \boxed{C_1e^{2t} + C_2e^{3t} - t - \frac{1}{2}}$[7] となる.

これを③に代入して $x' = 2C_1e^{2t} + 3C_2e^{3t} - 1$ だから

$y = 2C_1e^{2t} + 3C_2e^{3t} - 1 - 4C_1e^{2t} - 4C_2e^{3t} + 4t + 2 - t$

∴ $y = \boxed{-2C_1e^{2t} - C_2e^{3t} + 3t + 1}$[8] となる.

[1] 5 [2] 6 [3] $-6t + 2$ [4] 2 [5] 3

[6] $-t - \frac{1}{2}$

[7] $C_1e^{2t} + C_2e^{3t} - t - \frac{1}{2}$

[8] $-2C_1e^{2t} - C_2e^{3t} + 3t + 1$

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5 (1) $\Gamma(9) = 8! = 1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8 = \boxed{40320}$

(2) $\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{\pi}}{2}}$

(3) $\Gamma\left(\frac{7}{2}\right) = \frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \boxed{\frac{15}{8}\sqrt{\pi}}$

(4) $\Gamma\left(\frac{11}{2}\right) = \frac{9}{2}\cdot\frac{7}{2}\Gamma\left(\frac{7}{2}\right) = \frac{63}{4}\cdot\frac{15}{8}\sqrt{\pi} = \boxed{\frac{945}{32}\sqrt{\pi}}$ 3段

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6 (1) $B(2,2) = \frac{\Gamma(2)^2}{\Gamma(4)} = \frac{1^2}{3!} = \boxed{\frac{1}{6}}$

(2) $B\left(\frac{7}{2}, \frac{9}{2}\right) = \frac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{9}{2}\right)}{\Gamma(8)} = \frac{\frac{7}{2}\Gamma\left(\frac{7}{2}\right)^2}{7!} = \frac{\left(\frac{15}{8}\right)^2}{2\cdot6!}\pi$

$= \frac{5}{2^{11}}\pi = \boxed{\frac{5}{2048}\pi}$

(3) $I = \int_0^{\pi/2}\sin^{2\cdot\frac{7}{2}-1}x\cos^{2\cdot4-1}x\,dx = \frac{1}{2}B\left(\frac{7}{2}, 4\right)$

$= \frac{1}{2}\frac{\Gamma\left(\frac{7}{2}\right)\Gamma(4)}{\Gamma\left(\frac{15}{2}\right)} = \frac{1}{2}\frac{\Gamma\left(\frac{7}{2}\right)3!}{\frac{13}{2}\cdot\frac{11}{2}\cdot\frac{9}{2}\cdot\frac{7}{2}\Gamma\left(\frac{7}{2}\right)} = \boxed{\frac{16}{3003}}$ 3段

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7. $t = \frac{1}{1+x}$ より $dt = -\frac{1}{(1+x)^2}dx = -t^2dx$

∴ $dx = \boxed{-\frac{1}{t^2}}$[1]$dt$, $x = \frac{1}{t} - 1 = \frac{1-t}{t}$

$\frac{x^{3/2}}{(1+x)^4} = \left(\frac{1-t}{t}\right)^{3/2}t^4 = t^{\boxed{5/2}}$[2]$(1-t)^{\boxed{3/2}}$[3]

x	$0 \to \infty$
t	$1 \to 0$

∴ $I = \int_0^1 t^{5/2}(1-t)^{3/2}t^{-2}dt$

$I = \int_0^1 t^{\boxed{1/2}}$[4]$(1-t)^{\boxed{3/2}}$[5]$dt = B\left(\boxed{\frac{3}{2}}, \boxed{\frac{5}{2}}\right) = \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{5}{2}\right)}{\Gamma(\boxed{4})}$[7]

$= \frac{\frac{3}{2}\Gamma\left(\frac{3}{2}\right)^2}{3!} = \boxed{\frac{\pi}{16}}$[8]

[2] $\frac{5}{2}$ [3] $\frac{3}{2}$ [4] $\frac{1}{2}$ [5] $\frac{3}{2}$ [6] $\frac{5}{2}$ [7] 4 [8] $\frac{\pi}{16}$

4段

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8. $t = \sqrt{\log\frac{1}{x}}$ とおけば $-\log x = t^2$ ∴ $x = e^{-t^2}$

$dx = -2te^{-t^2}dt$ だから

x	$0 \to 1$
t	$\infty \to 0$

$I = 2\int_0^\infty t^2e^{-t^2}dt = \left[-te^{-t^2}\right]_0^\infty + \int_0^\infty e^{-t^2}dt$

$\lim_{t\to\infty}\frac{t}{e^{t^2}} = \lim_{t\to\infty}\frac{1}{2te^{t^2}} = 0$ だから

$I = \int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$ 5行

9. 変数分離形 $\frac{1}{N(N-P)}\frac{dN}{dt}=-k$

$$\int\frac{dN}{N(N-P)}=\frac{1}{P}\int\left(\frac{1}{N-P}-\frac{1}{N}\right)dN=\frac{1}{P}\log\left|\frac{N-P}{N}\right|$$

$$\therefore \log\left|\frac{N-P}{N}\right|=-kPt+C$$

$$\frac{N-P}{N}=Ce^{-kPt} \quad \therefore \frac{P}{N}=1-Ce^{-kPt}$$

$\therefore N=\frac{P}{1-Ce^{-kPt}}$, $N(0)=\frac{P}{1-C}=\alpha P$ より

$C=1-\frac{1}{\alpha}=\frac{\alpha-1}{\alpha}$ $\therefore N=\frac{\alpha P}{\alpha+(1-\alpha)e^{-kPt}}$ $\boxed{6行}$

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10 (1) $y'=\alpha x^{\alpha-1}$, $y''=\alpha(\alpha-1)x^{\alpha-2}$ 代入する

$\{\alpha(\alpha-1)-3\alpha+4\}x^{\alpha}=0$ $\therefore \alpha^2-4\alpha+4=0$ $\therefore y_1=\boxed{x^2}$

(2) $y_2'=u'y_1+uy_1'$, $y_2''=u''y_1+2u'y_1'+uy_1''$ より

$x^2(x^2u''+4xu')-3x^3u'=0$ $\therefore \boxed{x}^{[1]}u''+u'=0$ $(x\neq 0$ より$)$

$u'=v$ とおけば $\boxed{x}^{[1]}v'+v=0$ $\int\frac{dv}{v}=-\int\frac{dx}{x}+C_1$

$\therefore v=C_1\boxed{\frac{1}{x}}^{[2]}$, $C_1=1$ として $v=\boxed{\frac{1}{x}}^{[2]}$ $\therefore u=\int\frac{dx}{x}$

$u=\boxed{\log x}^{[3]}$ $\therefore y_2=uy_1=\boxed{x^2\log x}^{[4]}$ 3 解答

[1] x | [2] $\frac{1}{x}$ [3] $\log x$ [4] $x^2\log x$

(3) $W(y_1,y_2)=\begin{vmatrix} x^2 & x^2\log x \\ 2x & 2x\log x+x \end{vmatrix}=\boxed{x^3}$

(4) $t=\log x$ $(x=e^t)$ より $\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{1}{x}\frac{dy}{dt}$, 同様にして

$\frac{d^2y}{dx^2}=-\frac{1}{x^2}\frac{dy}{dt}+\frac{1}{x^2}\frac{d^2y}{dt^2}$ となるので これらを ①に代入して

$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+4y=0$ …② 特性方程式 $\lambda^2-4\lambda+4=0$ は

2重解 $\lambda=2$ をもつので ②の一般解は

$$y=(C_1+C_2t)e^{2t}$$

t をもとの x に戻せば

$y=(C_1+C_2\log x)x^2$ $(e^{2t}=(e^t)^2$ より$)$ 6行.