微積4 学年末 2019年度

1〜3：臨時2の問1,2,7

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4. (1) [1] $\lambda^2+1=0 \therefore \lambda=\pm i$ $\therefore \boxed{y = C_1\cos x + C_2 \sin x}$

[2] $L(y) = 3e^{ix}$, $\eta = 3\dfrac{1}{(D-i)(D+i)}e^{ix} = \dfrac{3}{2i}e^{ix}\int dx$

$= \dfrac{3x}{2i}(\cos x + i\sin x)$ $\therefore \boxed{y = C_1\cos x + C_2\sin x - \frac{3}{2}x\cos x}$

(2) [1] $\lambda^2+\lambda-6=0 \therefore \lambda=+2,-3$ $\therefore y=\boxed{C_1e^{2x}+C_2e^{-3x}}$

[2] $\eta = \dfrac{1}{(D+3)(D-2)}e^{-3x} = -\dfrac{1}{5}e^{-3x}\int dx = -\dfrac{x}{5}e^{-3x}$

$\therefore \boxed{y = C_1e^{2x}+C_2e^{-3x}-\frac{x}{5}e^{-3x}}$

(3) [1] $\lambda^2-2\lambda-3=0 \therefore \lambda=3,-1$ $\therefore \boxed{y=C_1e^{3x}+C_2e^{-x}}$

[2] 山辺： $-3-2D+D^2 \,)\, 3x-1$ 　商 $-x+1$；$3x+2$；-3　$\therefore \boxed{y=C_1e^{3x}+C_2e^{-x}-x+1}$

(4) $|A-\lambda E| = \lambda^2+4\lambda+3 = (\lambda+1)(\lambda+3)=0$ $\therefore \lambda_1=\boxed{-3}$ [1]

$\lambda_2=\boxed{-1}$ [2]

$\lambda_1: \begin{pmatrix}1&1\\1&1\end{pmatrix} \to x_1=\begin{pmatrix}1\\ \boxed{-1}\end{pmatrix}$ [3]

$\lambda_2: \begin{pmatrix}-1&1\\1&-1\end{pmatrix} \to x_2=\begin{pmatrix}1\\ \boxed{1}\end{pmatrix}$ [4]　$\begin{pmatrix}x\\y\end{pmatrix} = P\begin{pmatrix}X\\Y\end{pmatrix}$

$\to \dfrac{d}{dt}\begin{pmatrix}X\\Y\end{pmatrix} = \begin{pmatrix}\boxed{-3}&0\\0&\boxed{-1}\end{pmatrix}\begin{pmatrix}X\\Y\end{pmatrix} + \begin{pmatrix}\boxed{-e^{2t}}\\0\end{pmatrix}$ [5] [7]

$X'+3X = -e^{2t}$ [6]　$\dfrac{1}{D+3}(-e^{2t}) = -\dfrac{1}{5}e^{2t}$

$\therefore X = \boxed{-\frac{1}{5}e^{2t}} + C_1e^{\boxed{-3t}}$ [8] [9]

$Y'+Y=0 \therefore Y = C_2e^{\boxed{-t}}$ [10]

$\therefore \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1&1\\-1&1\end{pmatrix}\begin{pmatrix}X\\Y\end{pmatrix} = \begin{pmatrix}X+Y\\-X+Y\end{pmatrix}$

$\therefore x = \boxed{C_1e^{-3t}+C_2e^{-t}-\frac{1}{5}e^{2t}}$ [11]

$y = \boxed{-C_1e^{-3t}+C_2e^{-t}+\frac{1}{5}e^{2t}}$ [12]

5. $p=\dfrac{dy}{dx}$ とおけば $\dfrac{d^2y}{dx^2} = \dfrac{dp}{dx} = \dfrac{dp}{dy}\dfrac{dy}{dx} = p\dfrac{dp}{dy}$

原方程式に代入：$(y+1)p\dfrac{dp}{dy}+p^3=0 \to \dfrac{dp}{dy} = -\dfrac{p^2}{y+1}$

$\int\dfrac{1}{p^2}dy = -\int\dfrac{dy}{y+1}+C_1$, $\boxed{\frac{1}{p} = \log(y+1)+C_1}$ 3

$\dfrac{dy}{dx} = \dfrac{1}{\log(y+1)+C_1} \to \int(\log(y+1)+C_1)dy = \int dx + C_2$

$(y+1)\log(y+1) - \int dy + C_1y = x+C_2$ 5

$\therefore \boxed{(y+1)\log(y+1)+C_1y = x+C_2}$ $(C_1-1 \to C_1)$

(6分)

6. $y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$ (1), $y'' = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$ (2)

①に代入

$\sum_{n=2}^{\infty} n(n-1)a_nx^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_nx^n - \sum_{n=1}^{\infty}2na_nx^n + \sum_{n=0}^{\infty}6a_nx^n = 0$

$\therefore \sum_{n=2}^{\infty} n(n-1)a_nx^{n-2} = \sum_{n=0}^{\infty}(n^2+n-6)a_nx^n$ (3)

左辺 $\sum_{m=0}^{\infty}(m+2)(m+1)a_{m+2}x^m$ [4] $= \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n$ [5]　$(n+3)(n-2)$

$\therefore a_{n+2} = \dfrac{(n+3)(n-2)}{(n+2)(n+1)}a_n$ (6), $a_2 = \dfrac{3\cdot(-2)}{2\cdot 1}a_0 = -3a_0$ [7]

$y'(0)=a_1=0$ のとき $y = (1-3x^2)a_0$ (8)

(8分)

7. $y''+\omega^2y = e^{i\Omega x}$ と考える

$\omega\neq\Omega \Rightarrow \eta = I_m\left(\dfrac{1}{D^2+\omega^2}e^{i\Omega x}\right) = \boxed{\frac{1}{\omega^2-\Omega^2}\sin\Omega x}$　2

$\omega=\Omega \Rightarrow \eta = I_m\left(\dfrac{1}{(D-i\Omega)(D+i\Omega)}e^{i\Omega x}\right)$

$= I_m\left(\dfrac{1}{2i\Omega}e^{i\Omega x}\dfrac{1}{D}1\right) = I_m\left(-i\dfrac{x}{2\Omega}e^{i\Omega x}\right)$

$= \boxed{-\frac{x}{2\Omega}\cos\Omega x} = \boxed{-\frac{x}{2\omega}\cos\omega x}$

(5分)

3

OK

8. (1) $y' = \alpha e^{\alpha x}$, $y'' = \alpha^2e^{\alpha x}$ として代入

$\alpha^2(x+1) - \alpha(2x+3) + 2 = \alpha(\alpha-2)x + (\alpha-1)(\alpha-2) = 0$

$\therefore y_1 = \boxed{e^{2x}}$　1点

(2) $y'' - \dfrac{2x+3}{x+1}y' + \dfrac{2}{x+1}y = 0$ より [1] $= \boxed{\frac{2x+3}{x+1}}$ [1]

$\int\dfrac{2x+3}{x+1}dx = \int\left(2+\dfrac{1}{x+1}\right)dx = \boxed{2x+\log(x+1)}$ [2]

$\therefore \exp([2]) = \boxed{(x+1)e^{2x}}$ [3]

$y_2 = e^{2x}\int\dfrac{(x+1)e^{2x}}{e^{4x}}dx = e^{2x}\int(x+1)e^{-2x}dx$

$= e^{2x}\left\{-\frac{1}{2}(x+1)e^{-2x} - \frac{1}{4}e^{-2x}\right\} = \boxed{-\frac{x}{2}-\frac{3}{4}}$ [4]

(3) $\boxed{y = C_1e^{2x}+C_2(2x+3)}$　6段 (11点)

9. $W(y_1,y_2) = \begin{vmatrix}\sin x & \sin(x+\alpha)\\ \cos x & \cos(x+\alpha)\end{vmatrix}$

$= \sin x\cos(x+\alpha) - \cos x\sin(x+\alpha) = \sin(x-x-\alpha)$

$= -\sin\alpha$ $\therefore \alpha=0,\pi$ で 線形従属, $\alpha\neq 0,\pi$ で 線形独立.　4行(4点)

10. $t=\log x$ と置換 $\to \dfrac{dy}{dx} = \dfrac{dy}{dt}\cdot\dfrac{1}{x}$, $\dfrac{d^2y}{dx^2} = -\dfrac{1}{x^2}\dfrac{dy}{dt} + \dfrac{1}{x^2}\dfrac{d^2y}{dt^2}$

$= \dfrac{1}{x^2}\left(\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}\right)$ (2)　これらを①に代入すると①の

左辺 $= \dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - \dfrac{dy}{dt} + y = \boxed{\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y}$ (3)

右辺 $= \boxed{e^{2t}(3e^{2t}+2)}$ (4), $f(D) = \boxed{D^2-2D+1}$ (5)

$f(D)y=0 \to y_0 = \boxed{(C_1+C_2t)e^t}$ (6)

$\eta = \dfrac{1}{(D-1)^2}(3e^{4t}+2e^{2t}) = \boxed{\frac{1}{3}e^{4t}+2e^{2t}}$ (7)

$\therefore y = \boxed{(C_1+C_2\log x)x + \frac{1}{3}x^4 + 2x^2}$ (8)