D3 微積4 後期中間 2019

試験答案用紙　　　　番号　　　　氏名

4.

(1) $\frac{dy}{dt} = -ky$

(2) [1] $\frac{1}{y}y' = -1$, $\log|y| = -x + c$ $\therefore$ $y = ce^{-x}$

[2] $y = ue^{-x}$ $(u = u(x))$ として①に代入 $u'e^{-x} = e^{-x}$

$\therefore u = x + c$ $\therefore$ $y = (x+c)e^{-x}$

[3] $y(1) = (c+1)e^{-1} = 5$ $\therefore c = 5e - 1$

$\therefore$ $y = (x + 5e - 1)e^{-x}$

(3) [1] $y' = \frac{1}{x+c}$, $x + c = e^{y}$ $\therefore$ $y' = e^{-y}$

[2] $y' = -\frac{c}{(x+1)^2}$, $c = (x+1)y$ $\therefore$ $y' = -\frac{y}{x+1}$

(4) $\frac{1}{y}y' = \frac{2x}{x^2+1}$　$\log|y| = \log(x^2+1) + c$

$\therefore$ $y = c\boxed{(x^2+1)}$ [1] (x^2+1 は 1点)

$y = u(x)(x^2+1)$ として②に代入 $u'(x^2+1) = x(x^2+1)$

$\therefore$ $u' = \boxed{x}$ [2] $\therefore$ $u = \frac{1}{2}x^2$ [3] $+ c$ となるより

$y = \frac{1}{2}x^2(x^2+1) + c(x^2+1)$ [4]

$y(0) = 0$ である特殊解は $c = 0$ より $y = \frac{1}{2}x^2(x^2+1)$ [5]

(7点)

(5) [1] $u = \frac{y}{x}$ → $y = xu$ より $y' = u + xu'$

$\therefore u + xu' = u - \frac{2}{u}$　$xu' = -\frac{2}{u}$　$u' = -\frac{2}{xu}$

[2] $uu' = -\frac{2}{x}$　$\frac{1}{2}u^2 = -2\log x + c$

$\frac{y^2}{x^2} = c - 4\log x$ $\therefore$ $y^2 = cx^2 - 4x^2\log x$

[3] $y(1) = 3$ より　$9 = c$ $\therefore$ $y^2 = 9x^2 - 4x^2\log x$

1. (1) $\boxed{\frac{\pi}{2}}$ (2) $\boxed{\varepsilon}$ (3) $\boxed{\sqrt{5}}$ (4) $\boxed{r\cos\theta}$

(5) $\boxed{\cos\theta}$ (6) $\boxed{r}$ (7) $\boxed{\sin\theta}$ (8) $\boxed{\frac{1}{2}r^2}$

(9) $\boxed{5 - \varepsilon^2}$ (10) $\boxed{\frac{5}{2}}$

2. (1) $\boxed{\sqrt{3}}$ (2) $\frac{4\sqrt{2}}{3}\pi(8 - 3\sqrt{3})$

(3) $4\pi - \frac{64}{9}$ (4) $\boxed{\frac{81}{2}\pi}$

3. (1) $\frac{1}{8}(u - v)$ (2) $\frac{1}{2}(u + v)$ (3) $\boxed{\frac{1}{8}}$

(4) $\frac{1}{8}(5u + 3v)$ (5) $\boxed{5u + 3v}$

(6) $5uv + \frac{3}{2}v^2$ (7) $5u + 6$

(8) $\frac{5}{2}u^2 + 6u$ (9) $\boxed{4}$

(詳細は臨時1の解答例を参照)

5. $|z| \leqq R$ に対して D_z: $x^2 + y^2 \leqq R^2 - z^2$ とおくと

$\iint_{D_z} dxdy = \pi(R^2 - z^2)$ (1) 従って

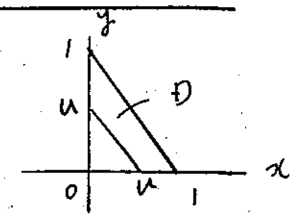
$I = \int_{-R}^{R}\left(\iint_{D_z} z^4\,dxdy\right)dz$ (2) $= \int_{-R}^{R} z^4\pi(R^2 - z^2)\,dz$ (3)

(3) $\pi(R^2z^4 - z^6)$

$= 2\pi\left[\frac{1}{5}R^2z^5 - \frac{1}{7}z^7\right]_0^R$ (4) $= \frac{4}{35}\pi R^7$ (5)　(3段)(5点)

6. (1) $x + y = u$, $y = uv$ $(x = u(1 - v))$

と変換すれば $D \to \tilde{D}$: $[0,1]\times[0,1]$



$J = \begin{vmatrix} 1-v & -u \\ v & u \end{vmatrix} = u$ より

左辺 $= \int_0^1\int_0^1 (u(1-v))^{p-1}(uv)^{q-1}(1-u)^{r-1}\cdot u\,dvdu$

$= \left(\int_0^1 u^{p+q-1}(1-u)^{r-1}du\right)\left(\int_0^1 v^{q-1}(1-v)^{p-1}dv\right)$　(8点)

$= B(p+q, r)B(p, q)$　(6行)

(2) 左辺 $= \frac{\Gamma(p+q)\Gamma(r)}{\Gamma(p+q+r)} \cdot \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} = \frac{\Gamma(r)\Gamma(p)\Gamma(q)}{\Gamma(p+q+r)}$

$= \frac{\Gamma(p)\Gamma(q+r)}{\Gamma(p+q+r)}\,\frac{\Gamma(q)\Gamma(r)}{\Gamma(q+r)} = B(p, q+r)B(q, r)$ (3行)

(4点)

7. $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ (1)

$\int_0^\infty xe^{-x^2}dx = \left[-\frac{1}{2}e^{-x^2}\right]_0^\infty$ (2) $= \boxed{\frac{1}{2}}$ (3)

$\int_0^\infty x^2e^{-x^2}dx = \left[-\frac{1}{2}xe^{-x^2}\right]_0^\infty$ (4) $+ \frac{1}{2}\int_0^\infty e^{-x^2}dx$ (5)

$= \boxed{\frac{\sqrt{\pi}}{4}}$ (6)　$\therefore$ $I = \frac{\sqrt{\pi}}{4} + 2\cdot\frac{1}{2} + \frac{\sqrt{\pi}}{2} = \frac{3}{4}\sqrt{\pi} + 1$ (7)

(4段)(8点)

8. (1) $y' + 4(y - 20) = 0$, $Y = y - 20$ とおけば

$Y' = y'$ より $Y' + 4Y = 0$ $\therefore Y = Ce^{-4t}$

$\therefore y = Ce^{-4t} + 20$ (3行) (4点)

(2) (1)で C を $u = u(t)$ に置き換えたものを

$y = ue^{-4t} + 20$ を ① に代入すると

$u'e^{-4t} = -100e^{-3t}$, $u' = -100e^{t}$

よって $u = -100e^{t} + C$ 以上より

$y = -100e^{-3t} + 20 + Ce^{-4t}$ (5行) (6点)

(3) $y(0) = -100 + 20 + C = 10$ $\therefore C = 90$

$y = -100e^{-3t} + 20 + 90e^{-4t}$ (12点) 2段

(4) $\lim_{t \to \infty} y = \boxed{20}$ (1点)

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9. $z = y^{-2}$ とおくと $z' = -2y^{-3}\, y'$ だから (1)

① より $-2y^{-3}y' - 2xy^{-2} = e^{x^2}$ $\therefore z' - 2xz = e^{x^2}$

$\therefore z' - \boxed{2x}\, z = -2e^{x^2}$ … ②
(2) (3)

$z' - 2xz = 0$ の一般解 $\frac{1}{z}z' = 2x$

$\log|z| = x^2 + C$ $\therefore z = C\boxed{e^{x^2}}$ (4)

C を $u = u(x)$ におきかえ ② に代入すると

$u'e^{x^2} = -2e^{x^2}$ $u' = -2$

$\therefore u = \boxed{-2x} + C$ (5) 従って

$z = \boxed{(C - 2x)e^{x^2}}$ (6) 以上より

(7)

$y^2 = \dfrac{1}{(C - 2x)e^{x^2}} = \boxed{\dfrac{e^{-x^2}}{C - 2x}}$

また $y = 0$ も解である。 6 (1段)