1. 半径を x, 高さを y, 一定の体積を V とすれば (1)

条件 $\pi x^2 y = V$ のもとで $f = 2\pi x^2 + 2\pi xy$ (2) 表面積

$\varphi = \pi x^2 y - V$ とおけば $\varphi_x = 2\pi xy$ (3), $\varphi_y = \pi x^2$ (4)

$f_x = 4\pi x + 2\pi y$ (5), $f_y = 2\pi x$ (6) より 未定乗数法から

$2x + y = \lambda xy$, $2x = x^2\lambda$ ∴ $\lambda = \frac{2}{x}$ (7)

∴ $2x + y = 2y$ ∴ $2x = y$ ∴ $\frac{y}{x} = 2$ (8)

(5点)

2. $f = y - \alpha x^2 - \frac{1}{\alpha}$ とおけば $f_\alpha = -x^2 + \frac{1}{\alpha^2}$

∴ $\begin{cases} y = \alpha x^2 + \frac{1}{\alpha} & \cdots ① \\ x^2 = \frac{1}{\alpha^2} & \cdots ② \end{cases}$ (1)(1点)

①^2

$y^2 = \alpha^2 x^4 + 2x^2 + \frac{1}{\alpha^2} \overset{②}{=} 4x^2$ (2) ∴ $y = \pm 2x$ (3)

(3点)

3 (1) $z_x = \frac{x}{\sqrt{x^2+y^2}}$, $z_y = \frac{y}{\sqrt{x^2+y^2}}$ $\left(z_x = \frac{x}{z},\ z_y = \frac{y}{z}\right)$

$z_{xx} = \frac{z - xz_x}{z^2} = \frac{z - \frac{x^2}{z}}{z^2} = \frac{y^2}{(\sqrt{x^2+y^2})^3}$ (1)

$z_{xy} = -\frac{x}{z^2} z_y = -\frac{xy}{(\sqrt{x^2+y^2})^3}$ (2), $z_{yy} = \frac{x^2}{(\sqrt{x^2+y^2})^3}$ (3)

(2) $z_x = -2xz$, $z_y = 2yz$ より

$z_{xx} = -2z - 2xz_x = -2z + 4x^2 z = (4x^2-2)e^{y^2-x^2}$ (1)

$z_{xy} = -2xz_y = -4xy\,e^{y^2-x^2}$ (2) $z_{yy} = (4y^2+2)e^{y^2-x^2}$ (3)

(4点)

4. $z_x = 3x^2 - 4y + 4$, $z_y = -4x - 2y$

∴ $\begin{cases} 3x^2 - 4y + 4 = 0 \\ 2x + y = 0 \end{cases}$ → $3x^2 + 8x + 4 = 0$, $(3x+2)(x+2) = 0$

∴ $(-2, 4)$, $\left(-\frac{2}{3}, \frac{4}{3}\right)$

(1点)

5. $f_x = 2xe^{\frac{y}{2}}$, $f_y = \left(\frac{1}{2}x^2 + \frac{1}{2}y + 1\right)e^{\frac{y}{2}}$ より $f_x = f_y = 0$

を解いて $x = 0$, $\frac{1}{2}x^2 + \frac{1}{2}y + 1 = 0$ ∴ $A(0, -2)$

$f_{xx} = 2e^{\frac{y}{2}}$, $f_{xy} = xe^{\frac{y}{2}}$, $f_{yy} = \left(\frac{1}{4}x^2 + \frac{1}{4}y + 1\right)e^{\frac{y}{2}}$

ただし $D = f_{xx}f_{yy} - f_{xy}^2$ の A における値は

$D(0,-2) = 2e^{-1}\cdot\frac{1}{2}e^{-1} = e^{-2} > 0$, また

$f_{xx}(0,-2) = 2e^{-1} > 0$ より f は $(0,-2)$ で極小値

$f(0,-2) = -2e^{-1}$ をとる。

(7点)

6. (1) $F = x + y + \log x + \log y$ とおく。$F_x = 1 + \frac{1}{x}$,

$F_y = 1 + \frac{1}{y}$ ∴ $\frac{dy}{dx} = -\frac{1+\frac{1}{x}}{1+\frac{1}{y}} = -\frac{\frac{x+1}{x}}{\frac{y+1}{y}} = -\frac{y(x+1)}{x(y+1)}$

(2) $G = xe^y - ye^x$ とおく。$G_x = e^y - ye^x$, $G_y = xe^y - e^x$

∴ $\frac{dy}{dx} = -\frac{e^y - ye^x}{xe^y - e^x} = \frac{ye^x - e^y}{xe^y - e^x}$

7. $F = x^3 + y^3 + z^3 - 3xyz$ とおく。$F_z = 3z^2 - 3xy$,

$F_x = 3x^2 - 3yz$, $F_y = 3y^2 - 3zx$ より

(1) $z_x = -\frac{x^2 - yz}{z^2 - xy}$ (2) $z_y = -\frac{y^2 - zx}{z^2 - xy}$

8. $\frac{4}{2} + \frac{9}{3} - \frac{z_0^2}{4} = 1$ より $z_0^2 = 16$, $z_0 = 4$ ($z_0 > 0$ より) (1)

ここで $F_x = x$ (2), $F_y = \frac{2}{3}y$ (3), $F_z = -\frac{z}{2}$ (4)

よって 点$(2, 3, 4)$ における偏微分係数は順に

2, 2, −2 とする。(5) (6) (7)

∴ $2(x-2) + 2(y-3) - 2(z-4) = 0$

$x + y - z - 1 = 0$ (18)

D3 微積3 前期末, 2019 (25点分は別紙臨時試験3)

試験答案用紙　　(2) 番号　　氏名

5. (1) $\int_0^1\int_0^{\sqrt{x}} xy\,dy\,dx = \frac{1}{2}\int_0^1 x\left[y^2\right]_0^{\sqrt{x}}dx = \frac{1}{2}\int_0^1 x^2dx$

$= \frac{1}{6}\left[x^3\right]_0^1 = \boxed{\frac{1}{6}}$

(2) $\int_0^2\int_0^{\sqrt{4-y^2}} xy^2\,dx\,dy = \frac{1}{2}\int_0^2\left[x^2\right]_0^{\sqrt{4-y^2}}y^2dy = \frac{1}{2}\int_0^2(4-y^2)y^2dy$

$= \frac{1}{2}\int_0^2(4y^2-y^4)dy = \frac{1}{2}\left[\frac{4}{3}y^3-\frac{1}{5}y^5\right]_0^2 = \boxed{\frac{32}{15}}$

(3) $\int_0^{\pi/2}\int_0^{\pi/2}\cos(x-y)\,dy\,dx = \int_0^{\pi/2}\left[-\sin(x-y)\right]_0^{\pi/2}dx$

$= \int_0^{\pi/2}\{\sin x - \sin(x-\pi/2)\}dx = \left[-\cos x + \cos(x-\pi/2)\right]_0^{\pi/2} = \boxed{2}$

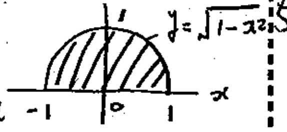
(4) $\int_0^1\int_1^2 e^{2x}e^y\,dy\,dx = \int_0^1 e^{2x}dx\int_1^2 e^y dy = \left[\frac{1}{2}e^{2x}\right]_0^1\left[e^y\right]_1^2$

$= \frac{1}{2}(e^2-1)(e^2-e) = \boxed{\frac{1}{2}e(e+1)(e-1)^2} = \frac{1}{2}(e^4-e^3-e^2+e)$

(5) $\int_1^3\int_x^{2x}\frac{y}{x}\,dy\,dx = \frac{1}{2}\int_1^3\frac{1}{x}\left[y^2\right]_x^{2x} = \frac{1}{2}\int_1^3(4x-x)dx$

$= \frac{1}{2}\int_1^3 3x\,dx = \frac{3}{4}\left[x^2\right]_1^3 = \boxed{6}$

(6) $\int_{-1}^1\int_0^{\sqrt{1-x^2}} x^2y\,dy\,dx = \frac{1}{2}\int_{-1}^1 x^2(1-x^2)dx$



5点

$= \int_0^1(x^2-x^4)dx = \left[\frac{1}{3}x^3-\frac{1}{5}x^5\right]_0^1 = \boxed{\frac{2}{15}}$

(7) $\int_0^2\int_{-2}^0(x^2+y^2)\,dy\,dx = \int_0^2\left[x^2y+\frac{1}{3}y^3\right]_{-2}^0dx$

$= \int_0^2\left(2x^2+\frac{8}{3}\right)dx = \left[\frac{2}{3}x^3+\frac{8}{3}x\right]_0^2 = \boxed{\frac{32}{3}}$

(8) $\int_0^{2\sqrt{3}}\frac{dx}{x^2+4}\cdot\int_0^2 y\,dy = \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^{2\sqrt{3}}\cdot\left[\frac{1}{2}y^2\right]_0^2$

$= \frac{1}{2}\tan^{-1}\sqrt{3}\cdot 2 = \boxed{\frac{\pi}{3}}$

(9) $\int_0^1\int_1^3(xy^2+y)\,dy\,dx = \int_0^1\left[\frac{1}{3}xy^3+\frac{1}{2}y^2\right]_1^3dx$

$= \int_0^1\left(\frac{26}{3}x+4\right)dx = \left[\frac{13}{3}x^2+4x\right]_0^1 = \boxed{\frac{25}{3}}$

6. $z_x = \boxed{f'(x+at)+f'(x-at)}$ [1], $z_t = \boxed{af'(x+at)-af'(x-at)}$ [2]

$z_{xx} = \boxed{f''(x+at)+f''(x-at)}$ [3], $z_{tt} = \boxed{a^2\{f''(x+at)+f''(x-at)\}}$ [4]

$\therefore\ z_{tt} = \boxed{a^2}z_{xx}$ [5]

7. (1) $z_x = 8x-2y^2$, $z_y = -4xy+5y^4$

$\therefore \begin{cases}4x = y^2\\ 4xy = 5y^4\end{cases}$ より $y^3 = 5y^4$ $\therefore \boxed{y=0,\ \frac{1}{5}}$

以上より停留点は $\boxed{(0,0),\ \left(\frac{1}{100},\frac{1}{5}\right)}$ (4行)

5点

(2) 原点をO, A$\left(\frac{1}{100},\frac{1}{5}\right)$とする. $\boxed{z_{xx}=8}$ …①

$\boxed{z_{yy} = -4x+20y^3}$, $\boxed{z_{xy} = -4y}$ だから $\boxed{H = z_{xx}z_{yy}-z_{xy}^2}$ とするから

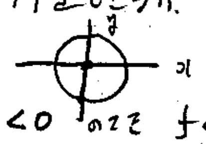
$H = 16(10y^3-2x-y^2)$

Aで $H = \frac{16}{50}\ \boxed{>0}$ で $\boxed{①から}$ Aにおいて f は $\boxed{極小値}$

$f\left(\frac{1}{100},\frac{1}{5}\right) = \boxed{-\frac{1}{12500}}$ をとる. これが求められる $\boxed{-3}$ Hが求められる 0点

Oで $H=0$ となるから H を用いた方法では判定できない.

$f(0,0)=0$ に注意すれば y軸 $(x=0)$ 上で



5点

$f(0,y) = y^5$ となるから $y>0$ のとき $f>0$, $y<0$ のとき $f<0$

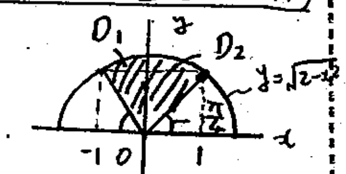
となるので $(0,0)$ のどのような近くの点でも $f(0,0)$ より大きい

値で小さい値をとる. よって $f(0,0)=0$ は極値でない.

12点 (10行)

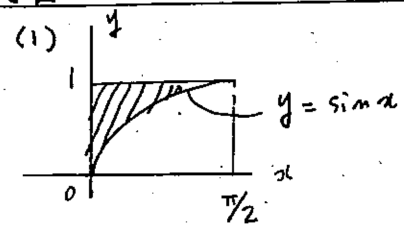
8. $D_1: -1\leqq x\leqq 0,\ -x\leqq y\leqq\sqrt{2-x^2}$

$D_2: 0\leqq x\leqq 1,\ x\leqq y\leqq\sqrt{2-x^2}$



とすれば $I = D_1+D_2$ の面積 $= \frac{1}{4}\cdot\pi\cdot(\sqrt{2})^2 = \boxed{\frac{\pi}{2}}$

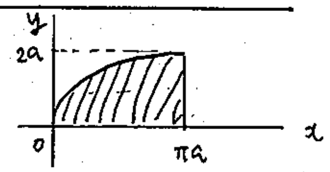
9. (1)



(2) $\int_0^1\int_0^{\sin^{-1}y} f(x,y)\,dx\,dy$

2段

10. (1) $I = \int_0^{\pi a}\int_0^y y\,dy\,dx$



$= \int_0^{\pi a}\boxed{\frac{1}{2}y^2}dx$ [1], $\frac{dx}{dt} = \boxed{a(1-\cos t)}>0$ [2] $(0<t<\pi)$ だから

$x = a(t-\sin t)$ と置換すれば $\begin{array}{c|c} x & 0\to\pi a\\ \hline t & 0\to\pi\end{array}$

2段 5点

与式 $= \boxed{\frac{1}{2}}\int_0^{\boxed{\pi}}\boxed{a^3(1-\cos t)^3}dt$ [3] [4]

(2) $1-\cos t = 2\sin^2\frac{t}{2}$ より

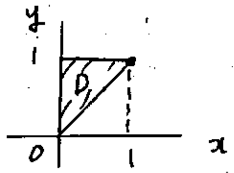
与式 $= 4a^3\int_0^{\pi}\sin^6\frac{t}{2}dt$, $\theta = \frac{t}{2}$ と置換して

$= 8a^3\int_0^{\pi/2}\sin^6\theta\,d\theta = 8a^3\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$

$= \frac{5}{4}\pi a^3$

4行

11. 積分領域を $D: 0\leqq x\leqq 1,\ x\leqq y\leqq 1$



与式 $= \iint_D e^{-y^2}dx\,dy = \int_0^1\int_0^y e^{-y^2}dx\,dy$

$= \int_0^1\left[x\right]_0^y e^{-y^2}dy = \int_0^1 ye^{-y^2}dy = \left[-\frac{1}{2}e^{-y^2}\right]_0^1$

$= \frac{1}{2}(1-e^{-1})$

4行

12. $F = x^4-2y^3-2x^2-3y^2+1$ とおく.

$F_x = 4x^3-4x$, $F_y = -6y^2-6y$, $F_{xx} = 12x^2-4$

$F_x = 0\to\boxed{x=0,\pm1}$ $x=0\to y=-1,\ \frac{1}{2}$

$x=\pm1\to y=0,\ -\frac{3}{2}$ $F_y\neq0$ から

$\left(0,\frac{1}{2}\right)$, $\left(\pm1,-\frac{3}{2}\right)$

$\left(0,\frac{1}{2}\right)$で $-\frac{F_{xx}}{F_y} = -\frac{8}{9}<0$ よって $x=0$ で極大値 $\frac{1}{2}$

$\left(\pm1,-\frac{3}{2}\right)$で $-\frac{F_{xx}}{F_y} = \frac{16}{9}>0$ よって $x=\pm1$ で極小値 $-\frac{3}{2}$

(6行)