D3 前中 2019 （微積3）

試験答案用紙　　　　番号　　　氏名

1.(1) [1] $(3-2i)e^{(3-2i)x}$　[2] $\frac{1}{2}(e^{ix}+e^{-ix}) = \cos x$

(2) [1] $\boxed{\frac{2}{3}}$　[2] $\lim_{n\to\infty} \frac{5n}{\sqrt{4n^2+5n}+2n} = \boxed{\frac{5}{4}}$

[3] $\frac{1}{1-\sqrt{2}} = -\sqrt{2}-1 < -1$ より 振動

[4] $\lim_{x\to+0} \frac{1-\cos x}{x^2} = \lim_{x\to+0} \frac{\sin x}{2x} = \boxed{\frac{1}{2}}$

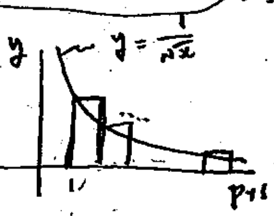
[5] $\lim_{x\to\infty} \frac{\log x}{\sqrt{x}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = \boxed{0}$

[6] $\lim_{n\to\infty} \frac{n^{100}}{90^n} \cdot \frac{(90^2)^n}{n!} = \boxed{0}$

(3) [1] $S_p = (1-\frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}) + \cdots + (\frac{1}{\sqrt{p}} - \frac{1}{\sqrt{p+1}})$
$= 1-\frac{1}{\sqrt{p+1}}$　[2] $S_p \to \boxed{1}$

(4) $\lim_{n\to\infty} \frac{\sqrt{n^2+1}-1}{n} = \lim_{n\to\infty}(\sqrt{1+1/n^2} - 1/n) = 1 \neq 0$ より

(5) ~~関~~ $f' = -\frac{1}{2x\sqrt{x}}$ [1]



$S_p > \int_1^{\boxed{p+1}\,[2]} f(x)dx = [\,2\sqrt{x}\,]_1^{p+1}$ [3]
$= 2\sqrt{p+1} - 2$ [4]

2.(1) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(2x)^{2n} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n}}{(2n)!} x^{2n}$

(2) $u = x+1$ とおくと $x = u-1$
$\therefore \frac{1}{x} = -\frac{1}{1-u} = -\sum_{n=0}^{\infty} u^n = -\sum_{n=0}^{\infty}(x+1)^n$ $(|x+1|<1)$

(3) $f' = 2xe^{-x} - x^2e^{-x} = x(2-x)e^{-x}$ [1]
$f'' = (2-2x-2x+x^2)e^{-x} = (2-4x+x^2)e^{-x}$ [2]
$f'=0 \iff x = \boxed{0}$ [3], $\boxed{2}$ [4], $f''(0) = \boxed{2}$ [5], $f''(2) = -\frac{2}{e^2}$ [6]
よって $f(0) = 0$ [7] は 極小値 [8], $f(2) = \frac{4}{e^2}$ [9] は 極大値 [10]　(5段)

(4) $e^x \fallingdotseq 1+x+\frac{x^2}{2}+\frac{x^3}{6}$ より $e = 1+1+\frac{1}{2}+\frac{1}{6} = \frac{16}{6} = \boxed{\frac{8}{3}}$

(5) [1] 与式 $=$ ~~$\cos n\pi + i\sin n\pi$~~ $= (e^{i\pi})^n = (-1)^n$
[2] 与式 ~~…~~
$\therefore$ 与式 $= (e^{i\frac{\pi}{2}})^n = i^n$

(6) [1] $f(x) = \frac{1}{x}\sin\frac{2x}{\pi}$ $(x>0)$ を考える
$\lim_{x\to+0} f(x) = \lim_{x\to+0} \frac{\frac{2}{\pi}\cos\frac{2x}{\pi}}{1} = \frac{2}{\pi}$　$\therefore \lim_{n\to\infty} n\sin\frac{2}{n\pi} = \frac{2}{\pi}$
[2] 与式 $= \lim_{n\to\infty} \frac{(2+1/n)(4-5/n)}{1/n^2 - 3} = -\frac{8}{3}$

(7) 与式 $= \lim_{n\to\infty} \frac{1}{n^2} \cdot \frac{n(n+1)}{2} = \frac{1}{2}$

(8) 与式 $= \sum_{n=0}^{\infty}(3\sqrt[3]{x})^n$ より $|3\sqrt[3]{x}| < 1$
$|x| < \left(\frac{1}{3}\right)^3 = \frac{1}{27}$
$\therefore \boxed{\frac{1}{27}}$

$|x| < \frac{1}{27}$ は収束域 ×

3.(1) [1] $\frac{dz}{dt} = \frac{1}{t}\frac{\partial z}{\partial x} + (1+t)e^t \frac{\partial z}{\partial y}$
[2] $\frac{dz}{dt} = (\cos t - \sin t)\frac{\partial z}{\partial x} + (\underbrace{\cos^2 t - \sin^2 t}_{\cos 2t})\frac{\partial z}{\partial y}$

(2) [1] $\frac{dz}{dt} = \left(\frac{1}{2\sqrt{t-1}} + \frac{1}{2\sqrt{t+1}}\right)\frac{1}{\sqrt{t-1}+\sqrt{t+1}}$
$= \frac{1}{2\sqrt{t-1}\sqrt{t+1}} = \frac{1}{2\sqrt{t^2-1}}$
[2] $\frac{dz}{dt} = \left(\frac{1}{t} - \frac{2}{t^2}\cdot 2\right)\cos\left(\log t + \frac{4}{t}\right)$
$= \frac{t-4}{t^2}\cos\left(\log t + \frac{4}{t}\right)$

(3) [1] $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\left(-\frac{t}{s^2}\right) + \frac{\partial z}{\partial y}\cdot 2s = -\frac{t}{s^2}\frac{\partial z}{\partial x} + 2s\frac{\partial z}{\partial y}$
[2] $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{1}{s} + \frac{\partial z}{\partial y}\cdot 2t = \frac{1}{s}\frac{\partial z}{\partial x} + 2t\frac{\partial z}{\partial y}$

(4) $z_x = 2x\log y$, $z_y = \frac{x^2}{y}$, $x_s = 2$, $x_t = 1$,
$y_s = t$, $y_t = s$ より
[1] $\frac{\partial z}{\partial s} = 4x\log y + t\frac{x^2}{y}$
$= 4(2s+t)\log st + \frac{(2s+t)^2}{s}$
[2] $\frac{\partial z}{\partial t} = 2x\log y + s\frac{x^2}{y}$
$= 2(2s+t)\log st + \frac{(2s+t)^2}{t}$

(5) $z = \log\sqrt{x^2} = \log|x|$ [1]　[2] $z$　[3] $\log x$
[4] $\lim_{r\to+0} \log r = -\infty$　~~2×4~~ 2×2

(6) [1] $z_x = \frac{4xy+3y^2}{2\sqrt{2x^2y+3xy^2}}$, $z_y = \frac{x^2+3xy}{\sqrt{2x^2y+3xy^2}}$
[2] $z_x = \frac{y}{(x+y)^2}$, $z_y = -\frac{x}{(x+y)^2}$
[3] $z_x = 3e^{3x}\tan 2y$, $z_y = \frac{2e^{3x}}{\cos^2 2y}$　2×4
[4] $z_x = 8x-3y$, $z_y = -3x+12y$

4. $f(0) = e^{1} = e$ [1], $f' = -e^{\cos x}\cdot \sin x$ [2]

$f'' = e^{\cos x}\sin^2 x - e^{\cos x}\cos x = e^{\cos x}(\sin^2 x - \cos x)$ [3]

$f^{(3)} = (-\sin^3 x + \sin x\cos x + 2\sin x\cos x + \sin x)\, e^{\cos x}$

$= (-\sin^3 x + 3\sin x\cos x + \sin x)\, e^{\cos x}$ [4]

$f^{(4)} = (\sin^4 x - 3\sin^2 x\cos x - \sin^2 x - 3\sin^2 x\cos x$
$+ 3\cos^2 x - 3\sin^2 x + \cos x)\, e^{\cos x}$

$= (\sin^4 x - 6\sin^2 x\cos x - 4\sin^2 x + 3\cos^2 x + \cos x)\, e^{\cos x}$ [5]

より, $f'(0) = 0$, $f''(0) = \boxed{-e}$ [6], $f^{(3)}(0) = 0$,

$f^{(4)}(0) = 4e$ [7] だから $f(x) = e - \dfrac{e}{2}x^2 + \dfrac{e}{6}x^4 + o(x^4)$ [8]

(6点)

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5. $z_x = \dfrac{1}{1+\left(\frac{2xy}{x^2-y^2}\right)^2} \cdot \dfrac{2y(x^2-y^2) - 4x^2y}{(x^2-y^2)^2}$

$= \dfrac{-2y(x^2+y^2)}{(x^2-y^2)^2 + 4x^2y^2} = \dfrac{-2y(x^2+y^2)}{(x^2+y^2)^2} = \dfrac{-2y}{x^2+y^2}$

$z_y = \dfrac{1}{1+\left(\frac{2xy}{x^2-y^2}\right)^2} \cdot \dfrac{2x(x^2-y^2) + 4xy^2}{(x^2-y^2)^2}$

$= \dfrac{2x(x^2+y^2)}{(x^2+y^2)^2} = \dfrac{2x}{x^2+y^2}$

3点ずつ
両方あったら
さらに1点
(6点)

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6. $f_x(0,0) = \lim_{h\to 0} \dfrac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0}\dfrac{h}{h} = 1$

$f_y(0,0) = \lim_{h\to 0} \dfrac{f(0,h) - f(0,0)}{h} = \lim_{h\to 0}\dfrac{-h}{h} = -1$ (2点)

$\varepsilon = f(x,y) - f(0,0) - x + y$　$(x,y) \neq (0,0)$ のとき

$= \dfrac{x^3-y^3}{x^2+y^2} - x + y = \dfrac{xy(x-y)}{x^2+y^2}$ (3点) (4点)

$y = 2x\ (x<0)$ のとき $\dfrac{\varepsilon}{\sqrt{x^2+y^2}} = -\dfrac{2}{5\sqrt{5}}\dfrac{x}{|x|} = \dfrac{2}{5\sqrt{5}}$

$x\to 0$ でも同じ $\dfrac{2}{5\sqrt{5}}$

(6点)