微積 2（学年末 2019年度）

試験答案用紙　　番号　　氏名

1. (1) $-\int \dfrac{dx}{x^2-(2\sqrt2)^2} = -\dfrac{1}{4\sqrt2}\log\left|\dfrac{x-2\sqrt2}{x+2\sqrt2}\right| = \dfrac{1}{4\sqrt2}\log\left|\dfrac{x+2\sqrt2}{x-2\sqrt2}\right|$

(2) $\dfrac12\left(x\sqrt{6-x^2}+6\sin^{-1}\dfrac{x}{\sqrt6}\right)$

(3) $\dfrac12\left(x\sqrt{x^2+9}+9\log\left(x+\sqrt{x^2+9}\right)\right)$

(4) $-\dfrac12\int(\cos 9x-\cos x)\,dx = \dfrac12\left(\sin x-\dfrac19\sin 9x\right)$

(5) $\int\dfrac{\cos x}{\cos^2 x}dx = \int\dfrac{dt}{1-t^2} = \dfrac12\log\left|\dfrac{1+t}{1-t}\right| = \dfrac12\log\dfrac{1+\sin x}{1-\sin x}$

$t=\sin x$

(6) $\int x^{-4}dx = -\dfrac13 x^{-3} = -\dfrac{1}{3x^3}$

(7) $\log|x|$

(8) $4\sin\dfrac{x}{4}$

(9) $\dfrac{1}{9+1}\cdot\dfrac19(9x+5)^{10} = \dfrac{1}{90}(9x+5)^{10}$

(10) $\tan x$

(11) $\log\left|x+\sqrt{x^2-9}\right|$

(12) $\sin^{-1}\dfrac{x}{\sqrt5}$

(13) $\int\left(1-\dfrac{10}{x^2+3}\right)dx = x-\dfrac{10}{\sqrt3}\tan^{-1}\dfrac{x}{\sqrt3}$

(14) $\dfrac{1}{25}e^{4x}(4\cos 3x+3\sin 3x)$

2. [1] $\left[-\dfrac{1}{3x^3}\right]_3^{\infty} = \dfrac{1}{81}$　[2] $\left[\dfrac32 x^{\frac23}\right]_0^8 = \dfrac32\cdot 4 = \boxed{6}$

(1段)

(2) 旅 $1\leqq t\leqq 2$ で $v\leqq 0$ だから移動した距離は

$\int_2^1 2\sin\pi t\,dt = \left[-\dfrac{2}{\pi}\cos\pi t\right]_2^1 = \dfrac{2}{\pi}(1+1) = \dfrac{4}{\pi}$

(3) $x' = \sec^2 t > 0$ より

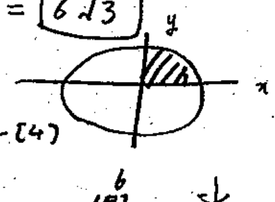
$\int_0^{\pi/4}(\sin t+1)\sec^2 t\,dt = \int_0^{\pi/4}\left(\dfrac{\sin t}{\cos^2 t}+\dfrac{1}{\cos^2 t}\right)dt$

$= \left[\dfrac{1}{\cos t}+\tan t\right]_0^{\pi/4} = \sqrt2+1-1 = \sqrt2$

(4) $x' = 6t,\ y' = 3-3t^2$ より $(x')^2+(y')^2 = 9(t^2+1)^2$

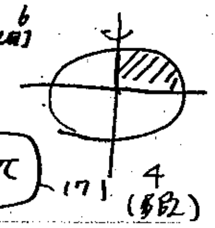
$\therefore\ l = \int_0^{\sqrt3}3(t^2+1)\,dt = 3\left[\dfrac13 t^3+t\right]_0^{\sqrt3} = 6\sqrt3$

(5) $x' = -8\sin t$ [1]　対称性から



$S = 4\int_0^{\boxed{\pi/2}}8\sin^2 t\,dt = 32\cdot\dfrac{\pi}{4} = 8\pi$ [2] [3] [4]

$V_x = 2\pi\int_0^{\pi/2}8\sin^3 t\,dt = 16\pi\cdot\dfrac23 = \dfrac{32}{3}\pi$ [5] [6]



$V_y = 2\pi\int_0^{\pi/2}64\cos^3 t\,dt = 128\pi\cdot\dfrac23 = \dfrac{256}{3}\pi$ [7]

4 (段)

(6) [1] $\left(3\cos\dfrac{\pi}{2},\ 3\sin\dfrac{\pi}{2}\right) = (0,3)$

[2] $\left(4\cos\dfrac43\pi,\ 4\sin\dfrac43\pi\right) = (-2,-2\sqrt3)$

(7) [1] $r = \sqrt{9+3} = 2\sqrt3$; $\cos\theta = -\dfrac{\sqrt3}{2}$, $\sin\theta = -\dfrac12$　$\therefore\ \theta = \dfrac76\pi$

$\therefore\ \left(2\sqrt3,\ \dfrac76\pi\right)$

[2] $r = 2\sqrt2$, $\cos\theta = -\dfrac{1}{\sqrt2}$, $\sin\theta = \dfrac{1}{\sqrt2}$　$\therefore\ \theta = \dfrac34\pi$　$\therefore\ \left(2\sqrt2,\ \dfrac34\pi\right)$

(8) 対称性から $S = 8\int_0^{\pi/4}\dfrac12\cos^2 2\theta\,d\theta = 4\cdot\dfrac12\int_0^{\pi/2}\cos^2 u\,du$

$= 2\cdot\dfrac{\pi}{4} = \dfrac{\pi}{2}$

(9) $l = \int_0^{\pi}\sqrt{e^{4\theta}+(2e^{2\theta})^2}\,d\theta = \sqrt5\int_0^{\pi}e^{2\theta}\,d\theta$

$= \sqrt5\left[\dfrac12 e^{2\theta}\right]_0^{\pi} = \dfrac{\sqrt5}{2}(e^{2\pi}-1) = \sqrt5\,e^{\pi}\cdot\sinh\pi$

3. $S = \dfrac12 OA\cdot OB\cdot\sin\theta$　($\theta:\angle AOB$)

$= \dfrac12 r_1 r_2\sin(\theta_2-\theta_1)$ (1)　　(1), (4) 2点

$\sin(\theta_2-\theta_1) = \sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1$ (2) だから

$S = \dfrac12(r_2\sin\theta_2\cdot r_1\cos\theta_1 - r_2\cos\theta_2\cdot r_1\sin\theta_1)$ (3)

$\therefore\ S = \dfrac12(x_1y_2 - x_2y_1)$ (4)

6点 (3段)

4. (1) $\dfrac{dN}{dt} = kN$

(2) $\dfrac1N\dfrac{dN}{dt} = k$, $\int\dfrac{dN}{N} = \int k\,dt+C$

$\log N = kt+C$　$\therefore\ N = e^c\cdot e^{kt}$

$t=0$ のとき $e^c e^0 = N_0$ より $e^c = N_0$　$\therefore\ N = N_0 e^{kt}$

4点

(3) $N_0e^{4k} = 10^4$, $N_0e^{6k} = 4\times10^4$ より

$e^{2k} = 4$, $e^k = 2$　$\therefore\ k = \log 2$

これと $N_0e^{4k} = 10^4$ から $(e^k)^4 N_0 = 10^4$　$\therefore\ 16N_0 = 10^4$

$\therefore\ N_0 = 625$

5点

5. $\cos x = \dfrac{1-t^2}{1+t^2}$ (1), $dx = \dfrac{2}{1+t^2}dt$ (2)

$\dfrac{1}{2+\cos x} = \dfrac{1}{2+\frac{1-t^2}{1+t^2}} = \dfrac{1+t^2}{3+t^2}$ (3)　$\therefore\ I = \int\dfrac{2}{3+t^2}dt$ (4)

$I = \dfrac{2}{\sqrt3}\tan^{-1}\dfrac{t}{\sqrt3}$ (5) $= \dfrac{2}{\sqrt3}\tan^{-1}\left(\dfrac{1}{\sqrt3}\tan\dfrac{x}{2}\right)$ (6)

6. (1) $\lim_{m\to\infty}\frac{1}{m}\sum_{k=1}^{m}\frac{1}{\sqrt{1+\frac{k}{m}}} = \int_0^1 \frac{dx}{\sqrt{1+x}} = \left[2\sqrt{1+x}\right]_0^1$

2点

$= 2(\sqrt{2}-1)$

(2) $\sum_{k=1}^{m}\frac{1}{\sqrt{2km-k^2}} = \frac{1}{m}\sum_{k=1}^{m}\frac{1}{\sqrt{2\cdot\frac{k}{m}-\left(\frac{k}{m}\right)^2}}$

$\therefore \lim_{m\to\infty}\sum_{k=1}^{m}\frac{1}{\sqrt{2km-k^2}} = \int_0^1\frac{dx}{\sqrt{2x-x^2}} = \int_0^1\frac{dx}{\sqrt{1-(x-1)^2}}$

(3行)

4点

$\underset{u=x-1}{=} \int_{-1}^{0}\frac{du}{\sqrt{1-u^2}} = \left[\sin^{-1}u\right]_{-1}^{0} = 0-\sin^{-1}(-1) = \frac{\pi}{2}$

7. $y' = 3x^2$ だから

$S = 2\pi\int_{-a}^{a}|x^3|\sqrt{1+9x^4}\,dx = 4\pi\int_0^a x^3\sqrt{1+9x^4}\,dx$

$= \frac{4\pi}{36}\int_0^a 36x^3\sqrt{1+9x^4}\,dx \underset{u=9x^4}{=} \frac{\pi}{9}\int_0^{9a^4}\sqrt{1+u}\,du$

$= \frac{\pi}{9}\left[\frac{2}{3}(1+u)^{\frac{3}{2}}\right]_0^{9a^4} = \frac{2}{27}\pi\left\{(1+9a^4)^{\frac{3}{2}}-1\right\}$ (4点)

8. $f' = (1-\cos x) \geqq 0$ (1) $\therefore f(x) \geqq f(0) = 0$

$\therefore \sin x \leqq x$ (1) $\therefore \sqrt{1-\frac{1}{2}x^2} \leqq \sqrt{1-\frac{1}{2}\sin^2 x} \leqq 1$

$\int_0^1\frac{dx}{\sqrt{1-\frac{1}{2}x^2}} \geqq \int_0^1\frac{dx}{\sqrt{1-\frac{1}{2}\sin^2 x}} \geqq \int_0^1 dx = 1$ …② (2)

$x = \sqrt{2}\sin\theta \to dx = \sqrt{2}\cos\theta\,d\theta$ (3), $\sqrt{1-\frac{1}{2}x^2} = \cos\theta$ (4)

だから $\int_0^1\frac{dx}{\sqrt{1-\frac{1}{2}x^2}} = \int_0^{\frac{\pi}{4}}\frac{\sqrt{2}\cos\theta}{\cos\theta}\,d\theta = \sqrt{2}\cdot\frac{\pi}{4}$ (5)(6)(7)

5点

4段

9. $I = \int_0^1 x^4(x(1-x))^{-\frac{1}{2}}dx = \int_0^1 x^{4-\frac{1}{2}}(1-x)^{-\frac{1}{2}}dx$

$= B\left(\frac{9}{2},\frac{1}{2}\right) = 2\int_0^{\frac{\pi}{2}}\sin^{2\cdot\frac{9}{2}-1}\theta\cdot\cos^{2\cdot\frac{1}{2}-1}\theta\,d\theta$

$= 2\int_0^{\frac{\pi}{2}}\sin^8\theta\,d\theta = 2\cdot\frac{7}{8}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$

$= \frac{35}{128}\pi$

(4点)

詳しく解くと $x = \sin^2\theta$ $\left(0\leqq\theta\leqq\frac{\pi}{2}\right)$ と置換

$I = \int_0^{\frac{\pi}{2}}(\sin^2\theta)^{\frac{7}{2}}(1-\sin^2\theta)^{-\frac{1}{2}}\cdot 2\sin\theta\cos\theta\,d\theta$

$= 2\int_0^{\frac{\pi}{2}}\sin^8\theta\,\frac{\cos\theta}{\cos\theta}\,d\theta$

(あとは同じ)