# C2 後中. 2019 (微積2：3章§1,2 4章§1)

試験答案用紙　$\frac{4-\sqrt7}{3}$　　番号　　氏名

1. (1) $\int_2^3 \sqrt{(x+1)^2-9}\,dx = \int_3^4 \sqrt{t^2-9}\,dt$
$= \left[\frac12\left(t\sqrt{t^2-9} - 9\log\left|t+\sqrt{t^2-9}\right|\right)\right]_3^4 = 2\sqrt7 + \frac92\log\frac{3}{4+\sqrt7}$

(2) $\int_{-1}^0 \frac{dx}{\sqrt{(x+1)^2+1}} = \int_0^1 \frac{dt}{\sqrt{t^2+1}} = \left[\log\left(t+\sqrt{t^2+1}\right)\right]_0^1$
$= \log\left(1+\sqrt2\right)$

(3) $(\tanh x)' = \frac{1}{\cosh^2 x}$ なので 与式 $= \left[\frac12\tanh\frac{x}{2}\right]_0^1$
$= \frac12\tanh\frac12 = \frac{e^{1/2}-e^{-1/2}}{2(e^{1/2}+e^{-1/2})} = \frac{e-1}{2(e+1)}$

(4) 与式 $= \left[-\frac15\cos^5 x\right]_0^{\pi/4} = \frac15\left(1-\frac{1}{4\sqrt2}\right) = \frac{8-\sqrt2}{40}$

(5) $\left[(x+3)e^x\right]_1^2 - \int_1^2 e^x\,dx = 5e^2-4e-e^2+e = 4e^2-3e$

(6) $\left[-(x+1)\cos x\right]_0^{\pi} + \int_0^{\pi}\cos x\,dx = \pi+1+1 = \pi+2$

(7) $\left[\frac12\left(x\sqrt{1-x^2}+\sin^{-1}x\right)\right]_0^{1/\sqrt2} = \frac14+\frac{\pi}{8} = \frac{2+\pi}{8}$

(8) 与式 $= 2\int_0^{\pi/2}\sin^7 t\,dt = 2\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3} = \frac{96}{105} = \frac{32}{35}$

(9) $\frac78\cdot\frac56\cdot\frac34\cdot\frac{\pi}{4} = \frac{35}{256}\pi = \frac{105}{768}\pi$

(10) $\left[\frac15x^5\log x\right]_1^e - \frac15\int_1^e x^4\,dx = \frac{e^5}{5} - \left[\frac1{25}x^5\right]_1^e$
$= \frac{e^5}{5}-\frac{e^5}{25}+\frac1{25} = \frac{4}{25}e^5+\frac1{25}$

(11) $(\log 2x)' = \frac{2}{2x} = \frac1x$ より 与式 $= \left[\log|\log 2x|\right]_e^{e^2}$
$= \log(\log 2e^2) - \log(\log 2e) = \log\frac{2+\log2}{1+\log2}$

2. (1) $-\frac16\log\left|\frac{x+3}{x-3}\right|$　~~(2) $\frac12(x\sqrt{x^2+2}$~~

(2) $\frac12\left(x\sqrt{x^2+2} + 2\log\left(x+\sqrt{x^2+2}\right)\right)$

(3) $\int(1-\cos^2x)^2\sin x\,dx = \int(1-2t^2+t^4)\cdot(-dt)$
$= -\cos x+\frac23\cos^3 x-\frac15\cos^5 x$

(4) $\frac12\int\frac{2e^x-2}{2e^x-2x-1}\,dx = \frac12\log\left(2e^x-2x-1\right)$

(5) 与式 $= \int\frac{dx}{\sin x\cos x+\cos^2 x} = \int\frac{1}{\tan x+1}\cdot\sec^2 x\,dx$
$= \int\frac{(\tan x+1)'}{\tan x+1}\,dx = \log|\tan x+1|$

(6) ($t=\cos x$) 与式 $= -\int\sqrt{t+2}\,dt = -\frac23(\cos x+2)^{3/2}$

(7) $-xe^{-x}+\int e^{-x}\,dx = -e^{-x}(x+1)$

(8) $\frac14x^2\sin4x - \frac12\int x\sin4x\,dx$
$= \frac14x^2\sin4x+\frac18x\cos4x-\frac18\int\cos4x\,dx$
$= \frac14x^2\sin4x+\frac18x\cos4x-\frac1{32}\sin4x$

(9) $\int\frac{x^2+2x+1}{x^2+1}\,dx = \int\left(1+\frac{2x}{x^2+1}\right)dx = x+\log(x^2+1)$

(10) $-\frac18\int\frac{dx}{x^2-1/8} = -\frac{\sqrt2}{8}\log\left|\frac{x-\frac1{2\sqrt2}}{x+\frac1{2\sqrt2}}\right|$

$\frac{1}{4\sqrt2}$　$a=\frac{1}{2\sqrt2}$, $2a=\frac{1}{\sqrt2}$

$= \frac{\sqrt2}{8}\log\left|\frac{2\sqrt2x+1}{2\sqrt2x-1}\right|$

$\sin\alpha\sin\beta = -\frac12\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}$
$\cos(-\theta)=\cos\theta$

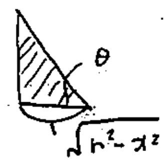
(11) $-\frac12\int(\cos11x-\cos7x)\,dx = \frac1{14}\sin7x-\frac1{22}\sin11x$

(12) $\frac{e^{3x}}{9+49}(3\sin7x-7\cos7x) = \frac{e^{3x}}{58}(3\sin7x-7\cos7x)$

(1つ3点)

3. (1) $\int_{14/9}^{23/3}\sqrt{1+\left(\frac32(x-1)^{1/2}\right)^2}\,dx = \frac12\int_{14/9}^{23/3}\sqrt{9x-5}\,dx$
$= \frac1{27}\left[(9x-5)^{3/2}\right]_{14/9}^{23/3} = \frac1{27}(8^3-3^3) = \frac{485}{27}$

(2) $\int_{-r}^{r}\frac12(r^2-x^2)\tan\theta\,dx$
$= \left[r^2x-\frac13x^3\right]_0^r\tan\theta = \frac23r^3\tan\theta$




(3) $\pi\int_0^{\pi/2}\cos^2 x\,dx = \pi\cdot\frac12\cdot\frac{\pi}{2} = \frac{\pi^2}{4}$

4. $t=\tan x$ とおく　$dt=\sec^2 x\,dx = (1+\tan^2 x)\,dx$
∴ $dt=(1+t^2)\,dx$ から $dx=\frac{1}{1+t^2}dt$ (1)　$\cos^2 x = \frac{1}{\sec^2 x} = \frac{1}{1+t^2}$
∴ $1+\sin^2 x = 2-\cos^2 x = 2-\frac{1}{1+t^2} = \frac{1+2t^2}{1+t^2}$ (2)
与式 $= \int\frac{t^2+1}{2t^2+1}\cdot\frac{1}{1+t^2}\,dt = \int\frac{1}{2t^2+1}\,dt$ (3)
$= \frac12\int\frac{dt}{t^2+\frac12} = \frac{\sqrt2}{2}\tan^{-1}\sqrt2\,t$ (4) $= \frac{\sqrt2}{2}\tan^{-1}(\sqrt2\tan x)$

(3段, 6点)

5. (1) $I_1 = \int\frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$

(2) $I_2 = \int\frac{dx}{a^2-x^2} = -\int\frac{dx}{x^2-a^2} = -\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$

(3) $I_{m+2} = \int\frac{dx}{\sqrt{(a^2-x^2)^{m+2}}} = \frac{1}{a^2}\int\frac{a^2-x^2+x^2}{\sqrt{(a^2-x^2)^{m+2}}}\,dx$
$= \frac{1}{a^2}I_m+\frac{1}{a^2}\int\frac{x^2}{\sqrt{(a^2-x^2)^{m+2}}}\,dx$ ---①
ここで $\int\frac{x}{\sqrt{(a^2-x^2)^{m+2}}}\,dx = -\frac12\int(-2x)(a^2-x^2)^{-1-\frac{m}{2}}\,dx$
$= -\frac12\cdot\left(-\frac{2}{m}\right)(a^2-x^2)^{-\frac{m}{2}} = \frac1m\frac{1}{\sqrt{(a^2-x^2)^m}}$ なから
部分積分法より ①のつづきは
$\frac{1}{a^2}I_m+\frac{1}{a^2}\left(\frac1m\frac{x}{\sqrt{(a^2-x^2)^m}} - \frac1m I_m\right)$
$= \frac{1}{a^2}\left(1-\frac1m\right)I_m + \frac{1}{ma^2}\frac{x}{\sqrt{(a^2-x^2)^m}}$
∴ $I_m = \frac{1}{ma^2}\left\{(m-1)I_m+\frac{x}{\sqrt{(a^2-x^2)^m}}\right\}$

7 (8行)

6. 接線の方程式を $y=g(x)$ とすれば 係数から曲線を $y=f(x)$ とすると $|f(x)-g(x)| = a(x-\alpha)^2|x-\beta|$
$\alpha<\beta$ のとき $\int_\alpha^\beta a(x-\alpha)^2(\beta-x)\,dx \underset{t=x-\alpha}{=} \int_0^{\beta-\alpha}a\,t^2((\beta-\alpha)-t)\,dt$
$= a\left[\frac13(\beta-\alpha)t^3-\frac14t^4\right]_0^{\beta-\alpha} = \frac1{12}a(\beta-\alpha)^4$
$\alpha>\beta$ のときは $\int_\beta^\alpha a(x-\alpha)^2(x-\beta)\,dx = \int_\alpha^\beta a(x-\alpha)^2(\beta-x)\,dx$
となり同じになる。

(6行)

7. $\dfrac{x}{x^3-1} = \dfrac{\boxed{\frac{1}{3}}}{x-1} + \dfrac{\boxed{-\frac{1}{3}}x + \boxed{\frac{1}{3}}}{x^2+\boxed{x+1}}$ [1] [3] [4] , $\displaystyle\int \frac{\frac{1}{3}}{x-1}dx = \boxed{\frac{1}{3}\log|x-1|}$ (5)

$\dfrac{-\frac{1}{3}x+\frac{1}{3}}{x^2+x+1} = \dfrac{-\frac{1}{3}\left(\boxed{x+\frac{1}{2}}\right)+\boxed{\frac{1}{2}}}{\left(\boxed{x+\frac{1}{2}}\right)^2+\frac{3}{4}}$ (6) (12) [7] (6)

$$\int \frac{-\frac{1}{3}(x+\frac{1}{2})}{(x+\frac{1}{2})^2+\frac{3}{4}}dx = -\frac{1}{6}\log\left((x+\tfrac{1}{2})^2+\tfrac{3}{4}\right)$$

$$= \boxed{-\frac{1}{6}\log(x^2+x+1)} \quad (8)$$

$$\int \frac{\frac{1}{2}}{(x+\frac{1}{2})^2+\frac{3}{4}}dx = \frac{1}{\sqrt{3}}\tan^{-1}\frac{2}{\sqrt{3}}(x+\tfrac{1}{2})$$

$$= \boxed{\frac{1}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}}(2x+1)} \quad (9)$$

$$I = \frac{1}{3}\log|x-1| - \frac{1}{6}\log(x^2+x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}$$

$$= \frac{1}{6}\log\boxed{\frac{(x-1)^2}{x^2+x+1}} + \boxed{\frac{1}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}} \quad (10)\ (9)$$

(7点) 12点 (8点)

---

8. (1) $\dfrac{7x^2+12x+2}{(x-2)(x+1)^3} = \dfrac{a}{x-2} + \dfrac{b}{x+1} + \dfrac{c}{(x+1)^2} + \dfrac{d}{(x+1)^3}$

として $a, b, c, d$ を求めると

$$\frac{2}{x-2} - \frac{2}{x+1} + \frac{1}{(x+1)^2} + \frac{1}{(x+1)^3}$$

$$\therefore I = 2\log|x-2| - 2\log|x+1| - \frac{1}{x+1} - \frac{1}{2(x+1)^2}$$

$$= 2\log\left|\frac{x-2}{x+1}\right| - \frac{2x+3}{2(x+1)^2} \quad (5点)$$

---

(2) $$\int\frac{dx}{x^2+2} = \frac{x}{x^2+2} - \int x\cdot\left(\frac{1}{x^2+2}\right)'dx$$

$= \dfrac{x}{x^2+2} + 2\displaystyle\int\frac{x^2}{(x^2+2)^2}dx$, 分子 $=(x^2+2)-2$ として

$$= \frac{x}{x^2+2} + 2\int\frac{dx}{x^2+2} - 4\int\frac{dx}{(x^2+2)^2}$$

$$\therefore \int\frac{dx}{(x^2+2)^2} = \frac{1}{4}\left(\frac{x}{x^2+2} + \int\frac{dx}{x^2+2}\right)$$

$$= \frac{x}{4(x^2+2)} + \frac{1}{4\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} \quad (5点)$$

5 (3) 別証明　1を利用した部分積分法を用いて $I_m$ から計算する.

$$I_m = \int(a^2-x^2)^{-\frac{m}{2}}dx = x(a^2-x^2)^{-m/2} - \int x\left((a^2-x^2)^{-m/2}\right)'dx$$

$$= x(a^2-x^2)^{-\frac{m}{2}} - m\int x^2(a^2-x^2)^{-\frac{m+2}{2}}dx$$

$$= x(a^2-x^2)^{-\frac{m}{2}} + m\int\frac{-x^2+a^2-a^2}{\sqrt{(a^2-x^2)^{m+2}}}dx$$

$$\therefore I_m = \frac{x}{\sqrt{(a^2-x^2)^m}} + mI_m - ma^2 I_{m+2}$$

$$\therefore ma^2 I_{m+2} = \frac{x}{\sqrt{(a^2-x^2)^m}} + (m-1)I_m$$