微積1 前中 2019

試験答案用紙　　　番号　　　氏名

1. (1) $y' = 6(3x^2-2)^5(6x) = 36x(3x^2-2)^5$

(2) $y' = -7(x^7-x)^{-8}(7x^6-1) = -\frac{7(7x^6-1)}{(x^7-x)^8}$

(3) $y' = \frac{2}{3}(x^2+x)^{-1/3}(2x+1) = \frac{2(2x+1)}{3\cdot\sqrt[3]{x^2+x}}$　(14段)

(4) $y' = -\frac{-\sin x}{(1+\cos x)^2} = \frac{\sin x}{(1+\cos x)^2}$

(5) $y' = -3\cos^2 x\sin x$

(6) $y' = \frac{3\cos 3x}{2\sqrt{\sin 3x}}$

(7) $y' = -2\sin x\cos x\cdot\exp(\cos^2 x)$

(8) $y' = \frac{2x+1}{x^2+x+1}$

(9) $y' = \cos x\log(\cos x) - \sin x\cdot\frac{\sin x}{\cos x}$

$= \cos x\log(\cos x) - \sin x\tan x$

指示に従う! ⇓ 通分し既約にする

(10) $y' = \{2\log(x+1) - \log x - \log(x-1)\}'$

$= \frac{2}{x+1} - \frac{1}{x} - \frac{1}{x-1} = \frac{1-3x}{x(x+1)(x-1)}$

(11) $y' = \left\{\frac{1}{3}\log(x^2+1) + \frac{3}{2}\log x\right\}'$

$= \frac{2x}{3(x^2+1)} + \frac{3}{2x} = \frac{13x^2+9}{6x(x^2+1)}$

(12) $y' = \sqrt{2}(\sqrt{2}x-3)^{\sqrt{2}-1}\sqrt{2} = 2(\sqrt{2}x-3)^{\sqrt{2}-1}$

(13) $y' = \frac{2x}{x^2-1}$

(14) $y' = \left\{\log_3(x-4) + \frac{1}{2}\log_3(x^2+3)\right\}'$

$= \frac{1}{\log 3}\left(\frac{1}{x-4} + \frac{x}{x^2+3}\right) = \frac{2x^2-4x+3}{(x-4)(x^2+3)\log 3}$

この行は不要

(15) $y' = \frac{3}{\sqrt{1-9x^2}}$　(16) $y' = -\frac{2x}{\sqrt{1-x^4}}$

(17) $y' = 2\cdot 3^{2x+3}\log 3$　(18) $y' = -5\sin(5x-1)$

(19) $y' = 2x\sin 4x + 4x^2\cos 4x$

$= 2x(\sin 4x + 2x\cos 4x)$

(20) $y' = 3e^{3x+2}$　(21) $y' = -4x^3 + \frac{1}{2}x$

(22) $y' = (x^2-1)' = 2x$

(23) $y' = (2x+1)(2x+1) + 2(x^2+x+1) = 6x^2+6x+3$

(24) $y' = \left(1 - \frac{2}{x^2+1}\right)' = \frac{4x}{(x^2+1)^2}$

(25) $y' = (x^3+1)' = 3x^2$

(26) $y' = \frac{3(x^2-5) - 2x(3x-1)}{(x^2-5)^2} = \frac{-3x^2+2x-15}{(x^2-5)^2}$

(27) $y' = -2x(2x-3)(x+2) + 2(1-x^2)(x+2) + (1-x^2)(2x-3) = -8x^3-3x^2+16x+1$

(28) $y' = -\frac{x^{-3}}{2} - \frac{5}{6}\cdot 3x^{-4} = -\frac{x+5}{2x^4}$　12 (14段)

2 (1) $\pi - \cos^{-1}\frac{1}{\sqrt{2}} = \frac{3}{4}\pi$　(2) $-\frac{\pi}{2}$

(3) $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$　(4) $-\frac{1}{2}$　(2段)

3. $x^2-x-2 = (x-2)(x+1)$ (1) ∴ $\lim_{x\to-1}(ax^2+bx+6) = 0$

$a-b+6=0$ より $b = a+6$ (2) このとき

$\lim_{x\to-1}\frac{ax^2+(a+6)x+6}{(x-2)(x+1)} = \lim_{x\to-1}\frac{(ax+6)(x+1)}{(x-2)(x+1)}$ (3)

$= \frac{6-a}{-3} = \frac{a-6}{3}$ (4) ∴ $a = 12$ [5] ∴ $b = 18$ [6]

(4分) (3段)

4. $f(0) = \sin\alpha$

$\lim_{x\to+0} f(x) = \lim_{x\to+0}\frac{x+3-3x-3}{x(\sqrt{x+3}+\sqrt{3x+3})}$

$= \lim_{x\to+0}\frac{-2}{\sqrt{x+3}+\sqrt{3x+3}} = \frac{-2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$\lim_{x\to-0} f(x) = \sin\alpha$ なので $\sin\alpha = -\frac{1}{\sqrt{3}}$　(5分)

∴ $\alpha =$　$\alpha = \sin^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\sin^{-1}\frac{1}{\sqrt{3}}$　(3段)

5. $\log y = 2\log|x| + \frac{1}{2}\log(1+x^2) - \frac{1}{2}\log(1-x^2)$　(1) [2] [3]

両辺をxで微分して

(3) $\frac{1}{y}y' = \frac{2}{x} + \frac{x}{1+x^2} + \frac{x}{1-x^2} = \frac{-2x^4+2x^2+2}{x(1+x^2)(1-x^2)}$　(4) (5) (6) (7)

以上より $\frac{dy}{dx} = y\,(7) = x^2\sqrt{\frac{1+x^2}{1-x^2}}\;\frac{-2x^4+2x^2+2}{x(1+x^2)(1-x^2)}$

$= 2x(1+x^2)^{-1/2}(1-x^2)^{-3/2}(1+x^2-x^4)$　[8] [9]　(6段)

$-x^4+x^2+1 = \left(\frac{1+\sqrt{5}}{2} - x^2\right)\left(x^2 + \frac{\sqrt{5}-1}{2}\right) > 0$　(10) [11] [12] $(|x|<1)$　(5段)

∴ $x>0$ のとき $\frac{dy}{dx}>0$　$x<0$ のとき $\frac{dy}{dx}<0$

(1) $|x|$　[2] $\frac{1}{2}$　(3) $\frac{1}{y}$　(4) $\frac{2}{x}$　(5) $\frac{x}{1+x^2}$

(6) $\frac{x}{1-x^2}$　(7) $\frac{-2x^4+2x^2+2}{x(1+x^2)(1-x^2)}$　[8] $-\frac{1}{2}$

[9] $-\frac{3}{2}$　(10) $1+x^2-x^4$　[11] $\frac{\sqrt{5}+1}{2}$

[12] $\frac{\sqrt{5}-1}{2}$

6.(1) 与式 $= \lim_{x\to 0} \dfrac{e^{ax}-1-(e^{bx}-1)}{x}$

$= \lim_{x\to 0} \left(\dfrac{e^{ax}-1}{x} - \dfrac{e^{bx}-1}{x} \right)$

$= \lim_{x\to 0} \left(a\cdot \dfrac{e^{ax}-1}{ax} - b\cdot \dfrac{e^{bx}-1}{bx} \right) = a-b$

(2) 与式 $= \lim_{x\to 0} \dfrac{e^{x\log 3} - e^{x\log 2}}{x}$

$= \lim_{x\to 0} \dfrac{e^{x\log 3}-1-(e^{x\log 2}-1)}{x}$

$= \lim_{x\to 0} \left(\dfrac{e^{x\log 3}-1}{x\log 3}\cdot \log 3 - \dfrac{e^{x\log 2}-1}{x\log 2}\cdot \log 2 \right)$

$= \log 3 - \log 2 = \log \dfrac{3}{2}$

(7分)

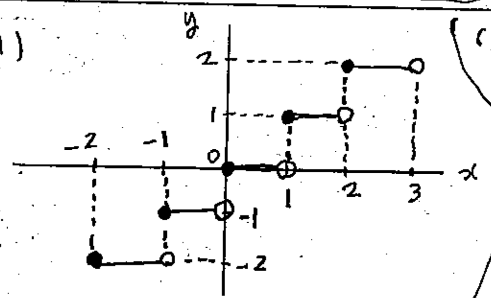
7. $f'(0) = \lim_{x\to 0} \dfrac{\frac{1}{\sin 3x} - \frac{1}{3x} - 0}{x} = \lim_{x\to 0} \dfrac{3x - \sin 3x}{3x^2 \sin 3x}$

$= \lim_{x\to 0} \dfrac{3x-\sin 3x}{(3x)^3} \cdot \dfrac{(3x)^3}{3x^2 \sin 3x}$

$= \lim_{x\to 0} \dfrac{3x-\sin 3x}{(3x)^3} \cdot \dfrac{3\cdot 3x}{\sin 3x} = \dfrac{1}{6}\cdot 3 = \dfrac{1}{2}$

(3分)

8 (1)



(2) [1] $\boxed{-2}$

[2] $\boxed{2}$

[3] $\boxed{1}$

[4] $\boxed{\text{極限値なし}}$

[5] $\boxed{n-1}$

4点

9. $f(x) = e^x - x - 2$ とおけば $-\infty < x < \infty$ で f は連続で $f(0) = 1-0-2 = -1 < 0$

$f(3) = e^3 - 5 > 2^3 - 5 = 3 > 0$

$x=0$ と $x=3$ で f が異符号だから中間値の定理より $f(x)=0$, 即 $e^x - x = 2$ の実数解が存在する

(5分)