C1 学末 2017 (基数I 第6章 §1)

2016年度試験答案用紙

$\frac{213}{3}$ 番号　　氏名

1. (1) [1] $\left(\frac{10-15}{5}, \frac{14-9}{5}\right) = \boxed{(-1,1)}$

[2] $\left(\frac{15-10}{5}, \frac{21-6}{5}\right) = \boxed{(1,3)}$

(2) [1] $\left(\frac{2+0+4}{3}, \frac{0+3+3}{3}\right) = \boxed{(2,2)}$　[2] 同様 $\boxed{\left(\frac{2}{3}, 1\right)}$

(3) $C(x,y)$ とすれば $\left(\frac{x+7}{3}, \frac{y+2}{3}\right) = (1,3)$ より $x=4, y=7$

$\therefore C\boxed{(-4,7)}$

(4) [1] 傾き $= -\frac{1}{3}$ より $y = -\frac{1}{3}(x-1)+2 \therefore \boxed{y = -\frac{x}{3} + \frac{7}{3}}$ $\boxed{x+3y-7=0}$

[2] $\boxed{x = \boxed{-\sqrt{2}}}$

[3] 傾き $= \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$ より $y = \frac{\sqrt{3}}{3}(x+2) \therefore \boxed{y = \frac{\sqrt{3}}{3}x + \frac{2}{\sqrt{3}}}$ $\boxed{\sqrt{3}x - 3y + 2\sqrt{3} = 0}$

[4] $y = 4(x+1)+2 \therefore \boxed{y = 4x+6}$

[5] $x+2y = d$ とする. $(1,3)$ を通るから $d=7 \therefore \boxed{x+2y-7=0}$

[6] $\boxed{y=3}$

[7] 交点は $y=-x$ を1式に代入して $(-1,1)$ ∴

$(x+1)+(y-1)=0 \quad \therefore \boxed{x+y=0}$

[8] 中点 $\left(\frac{5+1}{2}, \frac{1+3}{2}\right) = (3,2)$ で 線分の傾き $= -\frac{1}{2}$

だから $y = 2(x-3)+2 \therefore \boxed{y = 2x-4}$

(5) 求める点を (x,y) とすれば $x^2+y^2 = (x-1)^2+(y-2)^2$
$= (x+1)^2+(y-1)^2$

$\therefore \begin{cases} -2x-4y+5=0 \\ 2x-2y+2=0 \end{cases}$ より $(x,y) = \boxed{\left(\frac{1}{6}, \frac{7}{6}\right)}$

(6) $a:3:c = 3:4:-2$ だから $4a=9,\ 4c=-6$

[1] $\boxed{\frac{9}{4}}$　[2] $\boxed{-\frac{3}{2}}$

(7) $OA^2=16,\ OB^2 = x^2+y^2,\ AB^2 = (x-2)^2+(y-2\sqrt{3})^2$ より

$x^2+y^2=16,\ x^2-4x+y^2-4\sqrt{3}y=0 \ \therefore x+\sqrt{3}y=4$

これと再び $x^2+y^2=16$ を連立させて $\boxed{(4,0),\ (-2, 2\sqrt{3})}$

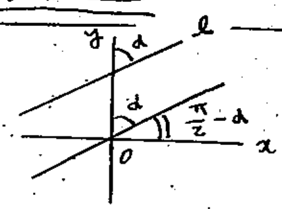
2 (1) $\left(\frac{-5\times15+2\times22}{2-5}, \frac{-5\times37-2\times14}{2-5}\right) = \left(\frac{31}{3}, \boxed{71}\right)$ $\xrightarrow{} \frac{213}{3}$

(2) $\left(\frac{-2\times15+3\times22}{3-2}, \frac{-2\times37-3\times14}{3-2}\right) = (36, -116)$

(3) $\left(\frac{-4\times15+7\times22}{7-4}, \frac{-4\times37-7\times14}{7-4}\right) = \left(\frac{94}{3}, \boxed{-82}\right) \to -\frac{246}{3}$

3. (1) l は x 軸との成す角が $\frac{\pi}{2}-\alpha$ で

あるから 傾き $= \tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha$



$\therefore \boxed{y = x\cot\alpha + h}$

(2) 垂直だから 傾きは $-\frac{1}{\cot\alpha} = -\tan\alpha$ となる. $\therefore l' : \boxed{y = -x\tan\alpha + h'}$

(3) $x\cot\alpha + h = -x\tan\alpha + h'$　$(\boxed{\cot\alpha+\tan\alpha})^{[1]}x = h'-h$

$\cot\alpha+\tan\alpha = \frac{\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\sin\alpha\cos\alpha} = \frac{2}{2\sin\alpha\cos\alpha} = \frac{\boxed{2}^{[2]}}{\sin 2\alpha}$

$\therefore x = \boxed{\frac{(h'-h)\sin 2\alpha}{2}}^{[3]}$　$\therefore y = (h'-h)\cos\alpha\sin\alpha\,\frac{\cos\alpha}{\sin\alpha}x + h$

$y = h'\boxed{\cos^2\alpha}^{[4]} + h(1-\cos^2\alpha) = h'\cos^2\alpha + h\boxed{\sin^2\alpha}^{[5]}$

$\boxed{\frac{\sin 2\alpha\cot\alpha}{2}}$ ×　[4][5] は 1つの三角関数が指示

(4) $y - \frac{h+h'}{2} = \frac{1}{2}(2y-h-h') = \frac{1}{2}(2h\sin^2\alpha + 2h'\cos^2\alpha - h - h')$

$= \frac{1}{2}\{h'(2\cos^2\alpha-1) - h(1-2\sin^2\alpha)\} = \frac{1}{2}(h'\cos 2\alpha - h\cos 2\alpha)$

$= \frac{1}{2}(h'-h)\cos 2\alpha$　よって 求める距離は

$\sqrt{\left\{\frac{1}{2}(h'-h)\sin 2\alpha\right\}^2 + \left\{\frac{1}{2}(h'-h)\cos 2\alpha\right\}^2} = \frac{1}{2}\sqrt{(h'-h)^2}$

$= \frac{1}{2}|h'-h|$

5行

4. (1) $a(x-1)+b(y-2)=0$ で 垂直条件から $3a+4b=0 \quad \therefore a=4, b=-3$

$4(x-1)-3(y-2)=0 \ \therefore \boxed{4x-3y+2=0}$　一般形指示

(2) $\begin{cases} 3x+4y=2 \\ 4x-3y=-2 \end{cases}$ から $x = -\frac{2}{25},\ y = \frac{14}{25} \quad \therefore \boxed{\left(-\frac{2}{25}, \frac{14}{25}\right)}$

5 (1) [1] $\boxed{A_1(-5,0)}$　[2] $\boxed{A_2(0,-3)}$

(2) $1:(k-6) = (k+7):2$ だから $(k-6)(k+7)=2$

$k^2+k-44=0 \quad \therefore k = \boxed{\frac{-1\pm\sqrt{177}}{2}}$

(3) $k+7+2(k-6)=0$ だから $3k=5 \quad \therefore k = \boxed{\frac{5}{3}}$

(4) (3) から $l_1 : x - \frac{13}{3}y + 5 = 0 \to 3x-13y+15=0$ …①

$l_2 : \frac{26}{3}x + 2y + 6 = 0 \to 13x+3y+9=0$ …②

①,② を連立させて解くと $x = \frac{-162}{178} = -\frac{81}{89},\ y = \frac{84}{89}$

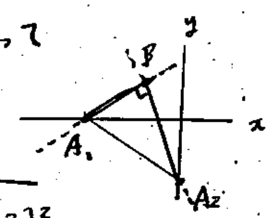
$\therefore B\left(-\frac{81}{89}, \frac{584}{89}\right)$

4行

(5) l_1 は A_1, B を通り, l_2 は A_2, B を通るから $\triangle A_1A_2B$

は直角三角形で $\angle A_1BA_2 = \frac{\pi}{2}$ となる. 従って

$S = \frac{1}{2}A_1B \times A_2B = \frac{1}{2}\sqrt{\left(5-\frac{81}{89}\right)^2+\left(\frac{84}{89}\right)^2}\sqrt{\left(\frac{81}{89}\right)^2+\left(\frac{84}{89}+3\right)^2}$



$= \frac{1}{2}\times\frac{4}{89}\times\sqrt{8722}\times\frac{9}{89}\times\sqrt{1602} = \frac{756}{89}$

4行

6 (1) 中点 $\boxed{\left(\frac{a-c}{2}, 0\right)}^{[1]}$, 傾き $\boxed{\frac{2b}{a+c}}^{[2]}$ より 求める直線の方程式

$y = -\frac{a+c}{2b}\left(x - \frac{a-c}{2}\right) \to y = -\frac{(a+c)x}{2b} + \frac{(a+c)(a-c)}{4b}$

$\boxed{2(a+c)}^{[3]}x + 4by + \boxed{c^2-a^2}^{[4]} = 0$　[3] : $(2a+2c) \to$ ○　$2a+2c \to$ ×　$-a^2+c^2 \to$ ×　$(-a^2+c^2) \to$ ○

(2) 中点 $\left(\frac{a+c}{2}, 0\right)$, 傾き $\frac{2b}{a-c}$ より 求める直線の方程式

$y = \frac{c-a}{2b}\left(x - \frac{a+c}{2}\right) = \frac{(c-a)x}{2b} - \frac{c^2-a^2}{4b}$

$\therefore \boxed{2(c-a)x - 4by + a^2 - c^2 = 0}$ (一般形指定)

(3) $\begin{array}{rl} & 2(a+c)x + 4by + c^2 - a^2 = 0 \\ + & 2(c-a)x - 4by + a^2 - c^2 = 0 \\ \hline & 4cx = 0 \end{array}$　$c \neq 0$ から $x=0,\ y = \frac{a^2-c^2}{4b}$

$\therefore \boxed{\left(0, \frac{a^2-c^2}{4b}\right)}$

(4) $\left(0, \frac{16-9}{8}\right) = \boxed{\left(0, \frac{7}{8}\right)}$

7(1) 内分点を求める公式から $a_n = \boxed{\frac{3}{8}}^{p} a_{n-1} + \boxed{\frac{5}{8}}^{q} b_{n-1} \cdots ①$

$b_n = \frac{5}{8} a_n + \frac{3}{8} b_{n-1} \underset{①}{=} \boxed{\frac{15}{64}}^{s} a_{n-1} + \boxed{\frac{49}{64}}^{t} b_{n-1} \cdots ②$

(2) (1) から $a_n - \alpha b_n = \frac{3}{8} a_{n-1} + \frac{5}{8} b_{n-1} - \alpha\left(\frac{15}{64} a_{n-1} + \frac{49}{64} b_{n-1}\right)$

$= \frac{3}{64}(8-5\alpha) a_{n-1} + \frac{1}{64}(40-49\alpha) b_{n-1}$, これが $r a_{n-1} - \alpha r b_{n-1}$

に等しいから $r = \frac{3}{64}(8-5\alpha) \cdots ③$, $\alpha r = \frac{1}{64}(49\alpha - 40) \cdots ④$

③,④から $24\alpha - 15\alpha^2 = 49\alpha - 40 \rightarrow 3\alpha^2 + 5\alpha - 8 = 0$

$(3\alpha+8)(\alpha-1) = 0 \quad \therefore \alpha = -\frac{8}{3}, r = 1 ; \alpha = 1, r = \frac{9}{64}$

(3) $\alpha = -\frac{8}{3}$ のとき $r = 1$ だから $a_n + \frac{8}{3} b_n$ は定数となり

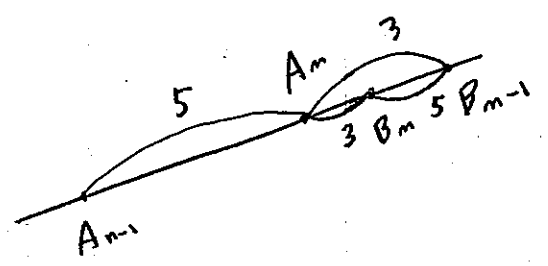
$a_n + \frac{8}{3} b_n = a + \frac{8}{3} b \cdots ⑤$, $\alpha = 1$ のとき $r = \frac{9}{64}$ だから

$a_n - b_n$ は等比数列となり $a_n - b_n = \left(\frac{9}{64}\right)^n (a-b) \cdots ⑥$

⑤-⑥から $b_n = \frac{3}{11}\left\{1 - \left(\frac{9}{64}\right)^n\right\} a + \frac{3}{11}\left\{\frac{8}{3} + \left(\frac{9}{64}\right)^n\right\} b$

これと⑥から $a_n = b_n + \left(\frac{9}{64}\right)^n (a-b)$

$= \frac{3}{11}\left\{1 + \frac{8}{3}\left(\frac{9}{64}\right)^n\right\} a + \frac{3}{11}\left\{\frac{8}{3} - \frac{8}{3}\left(\frac{9}{64}\right)^n\right\} b$



$\rightarrow P = \frac{1}{11}\left\{3 + 8\left(\frac{9}{64}\right)^n\right\}$, $Q = \frac{8}{11}\left\{1 - \left(\frac{9}{64}\right)^n\right\}$

$S = \frac{3}{11}\left\{1 - \left(\frac{9}{64}\right)^n\right\}$, $T = \frac{1}{11}\left\{8 + 3\left(\frac{9}{64}\right)^n\right\}$

8(1) 余弦定理から $PQ^2 = OP^2 + OQ^2 - 2OP \times OQ \cos\theta$

距離の公式から $\cos\theta = \frac{OP^2 + OQ^2 - PQ^2}{2OP \times OQ}$

$= \frac{1+m_1^2 + 1+m_2^2 - (m_2-m_1)^2}{2\sqrt{1+m_1^2}\sqrt{1+m_2^2}} = \frac{2 + 2m_1m_2}{2\sqrt{1+m_1^2}\sqrt{1+m_2^2}}$

$= \frac{1+m_1m_2}{\sqrt{1+m_1^2}\sqrt{1+m_2^2}}$

(2) (1) から $\cos\theta = \frac{1+\frac{2}{3}}{\sqrt{1+\frac{1}{9}}\sqrt{1+4}} = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{1}{\sqrt{2}}$

$\therefore \alpha = \theta = \boxed{\frac{\pi}{4}}$