Tables for the Quark-Hadron-Crossover 2018 (QHC18)
– Users' Manual v.1.0 (Jan.1, 2018) –

Abstract

This is a short note on the equation of state (EoS) tables, *Quark-Hadron-Crossover in 2018* (QHC18), which covers the EoS from the crust to quark matter domains. The QHC18 is a zero temperature EoS for neutron star matter in β-equilibrium. The physics behind QHC18 is discussed in the review paper (we call it the QHC18 paper),

"*From hadrons to quarks in neutron stars: a review*" (arXiv:1707.04966 [astro-ph.HE]),
G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song and T. Takatsuka,
(Reports on Progress in Physics, to be published).

This manual includes more remarks on numerics which may be important for some EoS users working on numerical simulations. The fitting forms of the numerical tables can be also found in Appendix C of the QHC18 review paper.

1. Tables

1.1. Thermodynamic quantities in the tables

The current version of the QHC equation of state (EoS), QHC18, includes numerical tables for

$$
\begin{aligned}
&1)\ \mu_B &&: \text{baryon chemical potential [MeV]}\\
&2)\ P &&: \text{pressure } [\text{MeV fm}^{-3}]\\
&3)\ n_B/n_0 &&: \text{baryon density/saturation density } (n_0 = 0.16\,\text{fm}^{-3})\\
&4)\ \varepsilon &&: \text{energy density } [\text{MeV fm}^{-3}]\\
&5)\ c_s/c &&: \text{sound velocity/light velocity}
\end{aligned} \tag{1}
$$

These tables are computed at $T = 0$ and β-equilibrium. The speed of sound was computed numerically using P and ε after preparing the tables for 1)-4).

1.2. The data files for QHC18 EoS

At present there are several versions of the QHC18 depending on the parameters (g_V, H) used for the quark matter EoS. We present our tables in text files eos_HQC18_gv***H***.txt

where *** indicates the values for (g_V, H). For instance, the quark EoS with the label "gv080H150" means $(g_V, H) = (0.80, 1.50)G_s$, where G_s is the scalar coupling of the Nambu–Jona-Lasinio (NJL) model for quark matter. The corresponding QHC18 EoS should be labelled as "QHC18-gv080H150".

Unless otherwise stated, we choose the "QHC18-gv080H150" as representative, and it will be simply called "QHC18". When we use other parameters, we will write "gv***H***" explicitly.

1.3. The data files for quark matter EoS

In addition to the tables of QHC18, we separately present the tables for quark matter EoS. The corresponding data files have the name

eos_Q18_gv***H***.txt

For the quark matter EoS tables, we cover the range of (g_V, H) for $g_V/G_s = 0.70, 0.80, 0.90, 1.00$ and $H/G_s = 1.40, 1.50, 1.60$. The range of the tables is from $n_B \simeq 4n_0$ to $\simeq 10n_0$.

1.4. Constants

In our tables we use natural units $c = \hbar = 1$. The constants used in our tables are

$$1\,\mathrm{fm}^{-1} = 197.326\,\mathrm{MeV}\,, \qquad \pi = 3.14159265\,. \tag{2}$$

The proton, neutron, electron, and muon masses are

$$m_p = 938.272\,\mathrm{MeV}\,, \qquad m_n = 939.565\,\mathrm{MeV}\,, \qquad m_e = 0.511\,\mathrm{MeV}\,, \qquad m_\mu = 105.6\,\mathrm{MeV}\,. \tag{3}$$

2. Unified equations of state: remarks on numerics (details)

The equations of state are divided into four distinct domains: the crust, nuclear liquid, hadron-quark crossover, and quark matter domains. For each domain we assign an equation of state as

$$
\begin{aligned}
\text{Crust}: &\quad \textbf{Togashi EoS} \quad [10^{-9}n_0 \le n_B \le 0.26n_0] \\
\text{Nuclear liquid}: &\quad \textbf{APR98 EoS} \quad [0.26n_0 \le n_B \le 2n_0] \\
\text{Crossover}: &\quad \textbf{QHC18} \quad [2n_0 \le n_B \le 5n_0] \\
\text{Quark matter}: &\quad \textbf{QHC18} \quad [5n_0 \le n_B \le 10n_0]
\end{aligned}
\tag{4}
$$

The description of each EoS is given in the following:

i) **Togashi EoS** for the crust is explained in details in the paper:

H. Togashi, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki and M. Takano,
Nucl. Phys. A 961, 78 (2017) ,

and the data tables can be found from the website [*http://user.numazu-ct.ac.jp/∼sumi/eos/#many-body*] where our manual is located. The reason we use this EoS is that this is a microscopic calculation consistent with the Akmar-Pandheripande-Ravenhall (APR) EoS (which we use for the nuclear liquid part), and therefore the crust and nuclear liquid parts join smoothly, in a way consistent with the thermodynamic relations. For the density below the neutron drip regime the EoS correctly reproduces the results obtained from laboratory nuclei.

The tables presented by these authors give μ_B, P, n_B/n_0, and ε. We use them as given in our EoS tables. The relation $P = \mu_B n_B - \varepsilon$ is satisfied to very good accuracy.

ii) **APR EoS** for the nuclear liquid is explained in the paper:

A. Akmal, V. R. Pandharipande and D. G. Ravenhall,
Phys. Rev. C **58** (1998) 1804 .

We created the data tables from the fit functions for ε_N (energy density for nuclear part) presented in this paper. The fit is given in the form $\varepsilon_N(n_B, x_p)$, where x_p is the proton fraction. The total energy is calculated by adding lepton contributions, $\varepsilon_l(n_B, x_p)$. The requirement of charge neutrality determines x_p as a function of n_B, so the total energy density is $\varepsilon(n_B) = \varepsilon_N(n_B, x_p(n_B)) + \varepsilon_l(n_B, x_p(n_B))$. From the analytic expression of $\varepsilon(n_B)$, one can calculate the analytic expression for $\mu_B = \partial\varepsilon(n_B)/\partial n_B$ from which the expression of P can be also derived by using the thermodynamic relation $P = n_B\mu_B - \varepsilon$. The numerical tables are obtained by substituting the values of n_B and $x(n_B)$ (determined numerically) into the analytic expressions for ε, μ_B, and P. Alternatively, we may take the numerical derivatives to evaluate μ_B and P, but the quality of the thermodynamic relations worsens.

The fitting function for APR contains the nucleon mass m_N. While we were not able to find out which values of m_N were used in their calculations, in creating our tables we used the average of the proton and neutron mass, $m_N = (m_p + m_n)/2 = 938.919\,\mathrm{MeV}$.

iii) **QHC18 EoS** for the hadron-quark crossover region:

The physics behind the crossover domain is explained in the QHC18 paper. To construct the EoS for the crossover domain, it is necessary to first construct the quark matter EoS at $n_B \geq 5n_0$ which will be explained below. After calculating a particular quark EoS, we interpolate the APR and quark matter EoS by polynomial functions for $P(\mu_B)$. We used the sixth order polynomials

$$P(\mu_B) = \sum_{n=0}^{5} c_n \mu_B^n \,. \tag{5}$$

From this analytic expression one can readily derive the analytic expression for n_B and ε as functions of μ_B. The coefficients c_n are determined by demanding that the crossover EoS match the APR and quark EoS's smoothly at $n_B = 2n_0$ and $5n_0$, up to the 2nd order derivatives. The tables were obtained by simultaneously substituting μ_B into the expressions of P, n_B, and ε.

iv) **QHC18 EoS** for the quark matter region:

The quark matter EoS in the QHC18 was calculated using the NJL model within the mean field approximation. As variable parameters, we choose g_V and H which quantify the strength of the repulsive density-density interaction and the attractive paring-interaction between quarks, respectively. As mentioned before, the quark EoS with the label "gv080H150" uses the parameter values $(g_V, H) = (0.80, 1.50)G_s$.

Firstly, the EoS is required to be stiff enough to be able to produce neutron stars of at least $2M_\odot$. Secondly the EoS must be such that, when used as the boundary condition at $n_B = 5n_0$ for the interpolation, the interpolating curve has a speed of sound $0 \leq c_s \leq 1$. These conditions severely restrict the ranges of (g_V, H).

There are analytic expressions for P, n_B, and ε as functions of μ_B, with a number of dynamical quantities related to the condensates which must be determined numerically. Once the condensates are fixed, all the thermodynamic variables are determined at once. This procedure yields the tables for the quark matter EoS.

3. Contact

We would appreciate it very much if you could give us suggestions on these EoSs. If you find any error or strange behavior, please contact

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