User Note for Relativistic EOS Table

(EOS2: 2010-version, with only nucleons)

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Abstract

This is a detailed note for users of the relativistic equation of state (EOS) of nuclear matter.

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1 Introduction

We write this document as a guide for the use of the relativistic equation of state (EOS) table. This EOS table is constructed in the following ranges:

- temperature T [MeV]: $-1.0 \leq \log_{10}(T) \leq 2.6$; mesh of $\log_{10}(T) = 0.04$

- proton fraction Y_p: $0.01 \leq Y_p \leq 0.65$; mesh of $Y_p = 0.01$

- baryon mass density ρ_B [g/cm³]: $5.1 \leq \log_{10}(\rho_B) \leq 16.0$; mesh of $\log_{10}(\rho_B) = 0.1$

We also add the results for the zero temperature case ($T = 0$) and the pure neutron matter case ($Y_p = 0$).

We have worked out consistent calculations for uniform matter and non-uniform matter in the relativistic mean-field (RMF) framework [1, 2]. We use the Thomas-Fermi approximation to describe inhomogeneous nuclear matter, which can be modeled as a mixture of free neutrons, free protons, alpha-particles, and a single species of heavy nuclei. For extremely low density and finite temperature, we approximate the nuclear matter as a classical ideal gas of free neutrons, free protons, and alpha-particles. Antiparticles have some contribution when temperature is very high. The thermodynamically favorable state is the one that minimizes the free energy density in this model, and we determine the most favorable state of nuclear matter at each temperature, proton fraction, and baryon mass density.

The leptons can be considered as uniform non-interacting relativistic particles, which are relatively easy to deal with. Hence, we provide the baryon EOS without the lepton contribution in this table. Users are supposed to add the lepton contribution to the baryon EOS.

This document is arranged as follows. In Sec. 2, we describe the RMF theory. In Sec. 3, we introduce the ideal-gas approximation to be used at low density. In Sec. 4, we describe the Thomas-Fermi approximation, which is used for calculating non-uniform matter. In Sec. 5, we make a detailed description of how to work out the EOS table. In Sec. 6, we list the definitions of physical quantities in the EOS table. Sec. 7 is devoted to suggestions and discussions for using the EOS table. Users who are not interested in the framework and calculating processes can only read Sec. 6 and Sec. 7 for the purpose to use the EOS table correctly.

2   Relativistic mean-field theory

We adopt the RMF theory with nonlinear σ and ω terms to describe homogeneous nuclear matter [3]. We start with the Lagrangian given by

$$
\begin{aligned}
\mathcal{L}_{\mathrm{RMF}} &= \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi \\
&\quad + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
&\quad - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\
&\quad - \frac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu}.
\end{aligned}
\tag{1}
$$

Here, ψ denotes an SU(2) baryon field of mass M (proton and neutron), and σ, ω^μ, and $\rho^{a\mu}$ are σ, ω, and ρ meson fields with masses m_σ, m_ω, and m_ρ, respectively. $W^{\mu\nu}$ and $R^{a\mu\nu}$ are the antisymmetric field tensors for ω^μ and $\rho^{a\mu}$, which can be written as

$$
W^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu,
\tag{2}
$$

$$
R^{a\mu\nu} = \partial^\mu\rho^{a\nu} - \partial^\nu\rho^{a\mu} + g_\rho\epsilon^{abc}\rho^{b\mu}\rho^{c\nu}.
\tag{3}
$$

In the Lagrangian, the constants g_σ, g_ω, and g_ρ are the coupling constants for the interactions between mesons and nucleons, the coefficients g_2 and g_3 are the self-coupling constants for the σ meson field, and c_3 is the self-coupling constant for the ω meson field. It is known that the inclusion of the nonlinear σ terms is essential to reproduce the properties of nuclei quantitatively and provide a reasonable value for the incompressibility, while the nonlinear ω term is added in order to reproduce the density dependence of the vector part of the nucleon self-energy obtained in the RBHF theory. The Lagrangian contains the meson masses, the coupling constants, and the self-coupling constants as parameters. We adopt the parameter set TM1 listed in Table 1, which was determined in Ref. [3]. With the TM1 parameter set, the symmetry energy is 36.9 MeV and the incompressibility is 281 MeV. The RMF theory with the TM1 parameter set provides excellent properties of heavy nuclei including unstable nuclei [3], and it is also shown to agree satisfactorily with experimental data in studies of nuclei with deformed configurations [4]. The properties of the EOS at high density with the TM1 parameter set were discussed in detail in Ref. [5], and the neutron star properties were studied using this EOS in Refs. [2, 5].

Table 1: The parameter set TM1 for the RMF Lagrangian

Parameter	TM1
M [MeV]	938.0
m_σ [MeV]	511.19777
m_ω [MeV]	783.0
m_ρ [MeV]	770.0
g_σ	10.02892
g_ω	12.61394
g_ρ	4.63219
g_2 [fm^{-1}]	-7.23247
g_3	0.61833
c_3	71.30747

Starting with the Lagrangian, we derive a set of Euler-Lagrange equations. The Dirac equation for the nucleon field is given by

$$(i\gamma_\mu \partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu})\,\psi = 0, \tag{4}$$

and the Klein-Gordon equations for the meson fields are given by

$$\partial_\nu\partial^\nu\sigma + m_\sigma^2\sigma = -g_\sigma\bar{\psi}\psi - g_2\sigma^2 - g_3\sigma^3, \tag{5}$$

$$\partial_\nu W^{\nu\mu} + m_\omega^2\omega^\mu = g_\omega\bar{\psi}\gamma^\mu\psi - c_3\left(\omega_\nu\omega^\nu\right)\omega^\mu, \tag{6}$$

$$\partial_\nu R^{a\nu\mu} + m_\rho^2\rho^{a\mu} = g_\rho\bar{\psi}\tau_a\gamma^\mu\psi + g_\rho\epsilon_{abc}\rho_\nu^b R^{c\nu\mu}. \tag{7}$$

These equations are coupled nonlinear quantum field equations, which are very difficult to solve exactly. We adopt the relativistic mean-field approximation as described in Ref. [6], in which the meson fields are treated as classical fields, and the field operators σ, ω^μ, and $\rho^{a\mu}$ are replaced by their expectation values $\langle\sigma\rangle$, $\langle\omega^\mu\rangle$, and $\langle\rho^{a\mu}\rangle$. In the present study, we first consider static infinite matter so that we obtain simplified equations, where the derivative terms in the Klein-Gordon equations vanish automatically due to the translational invariance of infinite matter. The spatial components of the vector meson fields vanish due to the rotational symmetry. For

the isovector-vector meson field $\rho^{a\mu}$, only the third isospin component has a non-vanishing value because of the charge conservation. Hence, the equations for the meson fields are reduced to

$$\sigma_0 \equiv \langle \sigma \rangle = -\frac{g_\sigma}{m_\sigma^2} \langle \bar{\psi}\psi \rangle - \frac{1}{m_\sigma^2} \left(g_2 \sigma_0^2 + g_3 \sigma_0^3 \right), \tag{8}$$

$$\omega_0 \equiv \langle \omega^0 \rangle = \frac{g_\omega}{m_\omega^2} \langle \bar{\psi}\gamma^0\psi \rangle - \frac{1}{m_\omega^2} c_3 \omega_0^3, \tag{9}$$

$$\rho_0 \equiv \langle \rho^{30} \rangle = \frac{g_\rho}{m_\rho^2} \langle \bar{\psi}\tau_3\gamma^0\psi \rangle. \tag{10}$$

The Dirac equation then becomes

$$\left(-i\alpha_k \nabla^k + \beta M^* + g_\omega \omega_0 + g_\rho \tau_3 \rho_0 \right) \psi_{is} = \varepsilon_{is} \psi_{is}, \tag{11}$$

where the index i denotes the isospin degree of freedom (proton and neutron), s denotes the index of eigenstates of nucleon, $M^* \equiv M + g_\sigma \sigma_0$ is the effective nucleon mass, and ε_{is} is the single-particle energy.

Nucleons occupy single-particle orbits with the occupation probability f_{is}. At zero temperature, $f_{is} = 1$ under the Fermi surface, while $f_{is} = 0$ above the Fermi surface. For finite temperature, the occupation probability is given by the Fermi-Dirac distribution,

$$f_{is} = \frac{1}{1 + \exp\left[(\varepsilon_{is} - \mu_i)/T \right]} = \frac{1}{1 + \exp\left[\left(\sqrt{k^2 + M^{*2}} - \nu_i \right)/T \right]}, \tag{12}$$

$$f_{\bar{i}s} = \frac{1}{1 + \exp\left[(-\varepsilon_{\bar{i}s} + \mu_i)/T \right]} = \frac{1}{1 + \exp\left[\left(\sqrt{k^2 + M^{*2}} + \nu_i \right)/T \right]}. \tag{13}$$

Here the indices i and $\bar{i}$ denote nucleons and antinucleons, and ε_{is} and $\varepsilon_{\bar{i}s}$ are the single-particle energy of nucleons and antinucleons, respectively. The relation between the chemical potential μ_i and the kinetic part of the chemical potential ν_i is give by

$$\mu_i = \nu_i + g_\omega \omega_0 + g_\rho \tau_3 \rho_0. \tag{14}$$

The chemical potential μ_i is related to the number density of nucleon n_i as

$$n_i = \frac{\gamma}{2\pi^2} \int_0^\infty dk \, k^2 \left(f_{ik} - f_{\bar{i}k} \right), \tag{15}$$

where γ is the degeneracy factor for the spin degree of freedom ($\gamma = 2$). The quantum number s is replaced by the momentum k when we do the integration in the momentum space instead

of summing over the eigenstates. The equations of the meson fields can be written as

$$\sigma_0 = -\frac{g_\sigma}{m_\sigma^2} \sum_i \frac{\gamma}{2\pi^2} \int_0^\infty dk\, k^2 \frac{M^*}{\sqrt{k^2 + M^{*2}}} \left(f_{ik} + f_{\bar{i}k}\right) - \frac{1}{m_\sigma^2} \left(g_2 \sigma_0^2 + g_3 \sigma_0^3\right), \tag{16}$$

$$\omega_0 = \frac{g_\omega}{m_\omega^2} \left(n_p + n_n\right) - \frac{c_3}{m_\omega^2} \omega_0^3, \tag{17}$$

$$\rho_0 = \frac{g_\rho}{m_\rho^2} \left(n_p - n_n\right). \tag{18}$$

Here, n_p and n_n are the proton and neutron number densities as defined in Eq. (15), and we denote $n_B = n_p + n_n$ as the baryon number density. We solve these equations self-consistently. The thermodynamical quantities are described in Ref. [6], and we simply write the expressions here. The energy density of nuclear matter is given by

$$\begin{aligned}
\epsilon =& \sum_i \frac{\gamma}{2\pi^2} \int_0^\infty dk\, k^2 \sqrt{k^2 + M^{*2}} \left(f_{ik} + f_{\bar{i}k}\right) + \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{3} g_2 \sigma_0^3 + \frac{1}{4} g_3 \sigma_0^4 \\
&+ g_\omega \omega_0 \left(n_p + n_n\right) - \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{4} c_3 \omega_0^4 \\
&+ g_\rho \rho_0 \left(n_p - n_n\right) - \frac{1}{2} m_\rho^2 \rho_0^2,
\end{aligned} \tag{19}$$

the pressure of nuclear matter is given by

$$\begin{aligned}
p =& \sum_i \frac{\gamma}{6\pi^2} \int_0^\infty dk\, k^2 \frac{k^2}{\sqrt{k^2 + M^{*2}}} \left(f_{ik} + f_{\bar{i}k}\right) - \frac{1}{2} m_\sigma^2 \sigma_0^2 - \frac{1}{3} g_2 \sigma_0^3 - \frac{1}{4} g_3 \sigma_0^4 \\
&+ \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{4} c_3 \omega_0^4 + \frac{1}{2} m_\rho^2 \rho_0^2,
\end{aligned} \tag{20}$$

and the entropy density is calculated by

$$\begin{aligned}
s =& \sum_i \frac{\gamma}{2\pi^2} \int_0^\infty dk\, k^2 \quad \left[-f_{ik} \ln f_{ik} - \left(1 - f_{ik}\right) \ln \left(1 - f_{ik}\right)\right. \\
&\left. - f_{\bar{i}k} \ln f_{\bar{i}k} - \left(1 - f_{\bar{i}k}\right) \ln \left(1 - f_{\bar{i}k}\right)\right].
\end{aligned} \tag{21}$$

3   Ideal-gas approximation

The interaction between nucleons is negligible at low density, so that we can treat nucleons as non-interacting Boltzmann particles at low density. We use the ideal-gas approximation to describe the mixed gas of neutrons, protons, and alpha-particles at low density and finite temperature. We note that the connection between the RMF results and the ideal-gas approximation is smooth.

For Boltzmann particles with spin 1/2, mass M, and number density n_i (i is n or p), we start with the partition function given by

$$Z_i = \left[(n_Q)^{n_i/2} / (n_i/2)! \right]^2 , \qquad (22)$$

where we have used the abbreviation $n_Q = \left(\frac{MT}{2\pi} \right)^{3/2}$. The factor $(n_i/2)!$ comes from avoiding the double counting for $n_i/2$ indistinguishable particles of spin up or spin down. We can calculate the free energy density from the partition function as

$$f_i = -T \ln Z_i = -T\, n_i \left[\ln(2n_Q/n_i) + 1 \right] . \qquad (23)$$

The entropy density is given by

$$s_i = -\left(\frac{\partial f_i}{\partial T} \right)_{n_i} = n_i \left[\ln(2n_Q/n_i) + \frac{5}{2} \right] , \qquad (24)$$

the internal energy density can be written as

$$\epsilon_i = f_i + T s_i = \frac{3}{2} T\, n_i, \qquad (25)$$

the chemical potential is obtained through the relation

$$\mu_i = \left(\frac{\partial f_i}{\partial n_i} \right)_T = -T \ln(2n_Q/n_i), \qquad (26)$$

and the pressure is given by

$$p_i = \left[n_i^2 \frac{\partial \left(f_i/n_i \right)}{\partial n_i} \right]_T = T\, n_i. \qquad (27)$$

The partition function of alpha-particles can be written as

$$Z_\alpha = \left[8n_Q \exp \left(B_\alpha/T \right) \right]^{n_\alpha} / n_\alpha!, \qquad (28)$$

where n_α is the alpha-particle number density, and $B_\alpha = 28.3$ MeV is the binding energy of an alpha-particle taken from Ref. [7]. The thermodynamical quantities of alpha-particles are given by

$$
\begin{aligned}
f_\alpha &= -T\, n_\alpha \left[\ln(8n_Q/n_\alpha) + 1 \right] - n_\alpha B_\alpha, & (29) \\
s_\alpha &= n_\alpha \left[\ln(8n_Q/n_\alpha) + \frac{5}{2} \right], & (30) \\
\epsilon_\alpha &= \frac{3}{2} T\, n_\alpha - n_\alpha B_\alpha, & (31) \\
\mu_\alpha &= -T \ln(8n_Q/n_\alpha) - B_\alpha, & (32) \\
p_\alpha &= T\, n_\alpha. & (33)
\end{aligned}
$$

For a mixed gas with the neutron number density n_n, the proton number density n_p, and the alpha-particle number density n_α, the free energy density is given by

$$f = f_n + f_p + f_\alpha. \tag{34}$$

We have to take into account the volume of the alpha-particle, otherwise the alpha-particle fraction would be some big number at high densities, where the alpha-particles should actually disappear. When we take into account the volume excluded by the alpha-particles, the free energy densities in Eq. (34) are given by

$$f_n = -(1-u)\,T\,\tilde{n}_n\,[\ln(2n_Q/\tilde{n}_n)+1]\,, \tag{35}$$

$$f_p = -(1-u)\,T\,\tilde{n}_p\,[\ln(2n_Q/\tilde{n}_p)+1]\,, \tag{36}$$

$$f_\alpha = -(1-u)\,\{T\,\tilde{n}_\alpha\,[\ln(8n_Q/\tilde{n}_\alpha)+1]-\tilde{n}_\alpha B_\alpha\}\,, \tag{37}$$

where $u = n_\alpha v_\alpha$ is the fraction of space occupied by alpha-particles with the effective volume of an alpha-particle $v_\alpha = 24\,\mathrm{fm}^{-3}$ taken from Ref. [7]. We denote the effective number densities of neutrons, protons, and alpha-particles as $\tilde{n}_n = n_n/(1-u)$, $\tilde{n}_p = n_p/(1-u)$, and $\tilde{n}_\alpha = n_\alpha/(1-u)$. The inclusion of the volume excluded by the alpha-particles has negligible effect in the low density region, while it is necessary for the calculation at high density. Actually, we just use Eqs. (35) and (36) for the case of $\tilde{n}_n + \tilde{n}_p < 10^{-5}\,\mathrm{fm}^{-3}$, while the RMF theory is used to describe nucleons when $\tilde{n}_n + \tilde{n}_p > 10^{-5}\,\mathrm{fm}^{-3}$. For alpha-particles, we use Eq. (37) to calculate the free energy in the whole density range. It is considered as a reasonable approximation because the number density of alpha-particles is not so large even though the total baryon density is high, and the alpha-particle fraction tends to vanish in the high density limit.

For very high temperature, we need to consider the contribution from antiparticles. The alpha-particle fraction is very small at high temperature, so we can treat the matter as uniform matter of nucleons and antinucleons. We have included the freedom of antiparticles in the RMF theory, but the RMF code meets some difficulty at extremely low density, so we need to take some approximation in order to describe the range of low density and high temperature. We take the nonrelativistic approximation of the RMF theory at low density, where the effective nucleon mass $M^* \approx M$, $< mesons > \approx 0$, and $\sqrt{k^2 + M^{*2}} \approx M + \frac{k^2}{2M}$. Then the occupation

probability of the Fermi-Dirac distribution becomes

$$f_{is} = AC_i \exp\left(-k^2/2MT\right), \tag{38}$$

$$f_{\bar{i}s} = \frac{A}{C_i} \exp\left(-k^2/2MT\right), \tag{39}$$

where $A = \exp\left(-M/T\right)$, and $C_i = \exp\left(\mu_i/T\right)$ with μ_i being the chemical potential of protons $(i = p)$ or neutrons $(i = n)$, which is related to the number density n_i as

$$n_i = 2A\left(C_i - \frac{1}{C_i}\right)\left(\frac{MT}{2\pi}\right)^{3/2}. \tag{40}$$

The thermodynamical quantities can be approximated as

$$\epsilon_i = 2A\left(C_i + \frac{1}{C_i}\right)\left(\frac{MT}{2\pi}\right)^{3/2}\left(\frac{3}{2}T\right), \tag{41}$$

$$p_i = 2A\left(C_i + \frac{1}{C_i}\right)\left(\frac{MT}{2\pi}\right)^{3/2} T, \tag{42}$$

$$s_i = 2A\left[\frac{5}{2}\left(C_i + \frac{1}{C_i}\right) - C_i \ln\left(AC_i\right) - \frac{1}{C_i}\ln\left(\frac{A}{C_i}\right)\right]\left(\frac{MT}{2\pi}\right)^{3/2}. \tag{43}$$

The contribution from antiparticles is negligible when $C_i \gg 1/C_i$, where the above expressions agree with the ideal-gas approximation.

4   Thomas-Fermi approximation

In the range, $T < 14$ MeV and $\rho_B < 10^{14.2}$ g/cm^3, where heavy nuclei may be formed in order to lower the free energy, we perform the Thomas-Fermi calculation based on the work done by Oyamatsu [8]. In this case, the non-uniform matter can be modeled as a mixture of free neutrons, free protons, alpha-particles, and a single species of heavy nuclei, while the leptons can be treated as uniform non-interacting particles separately. For the system with fixed proton fraction, the leptons play no role in the minimization of the free energy. Hence we mainly pay attention to baryon contribution in this calculation. Hereafter, we will not mention the leptons frequently, while we should keep in mind that there exists a uniform lepton gas everywhere.

We assume that each heavy spherical nucleus is located in the center of a charge-neutral cell consisting of a vapor of neutrons, protons, and alpha-particles. The nuclei form a body-centered-cubic (BCC) lattice to minimize the Coulomb lattice energy. It is useful to introduce

the Wigner-Seitz cell to simplify the energy of a unit cell. The Wigner-Seitz cell is a sphere whose volume is the same as the unit cell in the BCC lattice. We define the lattice constant a as the cube root of the cell volume. Then, we have

$$V_{\text{cell}} = a^3 = N_B/n_B, \tag{44}$$

where N_B and n_B are the baryon number per cell and the average baryon number density, respectively. We define the baryon mass density as $\rho_B = m_u n_B$ with m_u being the atomic mass unit. We calculate the Coulomb energy using this Wigner-Seitz approximation and add an energy correction for the BCC lattice [8]. This energy correction is negligible unless the nuclear size is comparable to the cell size.

We assume the nucleon distribution function $n_i(r)$ ($i = n$ for neutron, $i = p$ for proton) in the Wigner-Seitz cell as

$$n_i(r) = \begin{cases} \left(n_i^{\text{in}} - n_i^{\text{out}}\right)\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{\text{out}}, & 0 \le r \le R_i, \\ n_i^{\text{out}}, & R_i \le r \le R_{\text{cell}}, \end{cases} \tag{45}$$

where r represents the distance from the center of the nucleus and R_{cell} is the radius of the Wigner-Seitz cell defined by the relation

$$V_{\text{cell}} \equiv \frac{4\pi}{3} R_{\text{cell}}^3. \tag{46}$$

The density parameters n_i^{in} and n_i^{out} are the densities at $r = 0$ and $r \ge R_i$, respectively. The parameters R_i and t_i determine the boundary and the relative surface thickness of the heavy nucleus.

For the distribution function of alpha-particle $n_\alpha(r)$, which should decrease as r approaches the center of the heavy nucleus, we assume

$$n_\alpha(r) = \begin{cases} -n_\alpha^{\text{out}}\left[1 - \left(\frac{r}{R_p}\right)^{t_p}\right]^3 + n_\alpha^{\text{out}}, & 0 \le r \le R_p, \\ n_\alpha^{\text{out}}, & R_p \le r \le R_{\text{cell}}, \end{cases} \tag{47}$$

which could give $n_\alpha(r = 0) = 0$ and $n_\alpha(r > R_p) = n_\alpha^{\text{out}}$. Here we use the same parameters R_p and t_p for both proton and alpha-particle distribution functions in order to avoid the presence of too many parameters in the minimization procedure. The parameters R_n and t_n may be somewhat different from R_p and t_p due to the additional neutrons forming a neutron skin in the

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surface region. For a system with fixed temperature T, proton fraction Y_p, and baryon mass density ρ_B, there are eight independent parameters among the ten variables, a, n_n^{in}, n_n^{out}, R_n, t_n, n_p^{in}, n_p^{out}, R_p, t_p, and n_α^{out}. The thermodynamically favorable state is the one that minimizes the free energy density with respect to these eight independent parameters.

In this model the free energy density contributed from baryons is given by

$$f = F_{\text{cell}} / a^3 = (E_{\text{cell}} - T S_{\text{cell}}) / a^3, \tag{48}$$

where the free energy per cell F_{cell} can be written as

$$F_{\text{cell}} = (E_{\text{bulk}} + E_s + E_C) - T S_{\text{cell}} = F_{\text{bulk}} + E_s + E_C. \tag{49}$$

The bulk energy E_{bulk}, entropy S_{cell}, and bulk free energy F_{bulk} are calculated by

$$E_{\text{bulk}} = \int_{\text{cell}} \epsilon\left(n_n(r), n_p(r), n_\alpha(r)\right) d^3r, \tag{50}$$

$$S_{\text{cell}} = \int_{\text{cell}} s\left(n_n(r), n_p(r), n_\alpha(r)\right) d^3r, \tag{51}$$

$$F_{\text{bulk}} = \int_{\text{cell}} f\left(n_n(r), n_p(r), n_\alpha(r)\right) d^3r. \tag{52}$$

Here $\epsilon\left(n_n(r), n_p(r), n_\alpha(r)\right)$, $s\left(n_n(r), n_p(r), n_\alpha(r)\right)$, and $f\left(n_n(r), n_p(r), n_\alpha(r)\right)$ are the local energy density, entropy density, and free energy density at the radius r, where the system can be considered as a mixed uniform matter of neutrons, protons, and alpha-particles. Note that these local densities at each radius are calculated by treating neutrons and protons in the RMF theory as described in Sec. 2 for the case of $n_n(r) + n_p(r) > 10^{-5}\,\text{fm}^{-3}$, while we can use the method described in Sec. 3 for low density case of $n_n(r) + n_p(r) < 10^{-5}\,\text{fm}^{-3}$.

As for the surface energy term E_s due to the inhomogeneity of the nucleon distribution, we take the simple form

$$E_s = \int_{\text{cell}} F_0 \mid \nabla\left(n_n(r) + n_p(r)\right)\mid^2 d^3r. \tag{53}$$

The parameter $F_0 = 70\,\text{MeV} \cdot \text{fm}^5$ is determined by performing the Thomas-Fermi calculations for finite nuclei so as to reproduce the gross properties of the nuclear mass, charge radii, and the beta stability line as described in the Appendix of Ref. [8].

The Coulomb energy per cell E_C is calculated using the Wigner-Seitz approximation with an added correction term for the BCC lattice [8]

$$E_C = \frac{1}{2}\int_{\text{cell}} e\left[n_p(r) + 2n_\alpha(r) - n_e\right]\phi(r)d^3r + \triangle E_C, \tag{54}$$

where $\phi(r)$ represents the electrostatic potential calculated in the Wigner-Seitz approximation, n_e is the electron number density of a uniform electron gas ($n_e = Y_p n_B$), and $\triangle E_C$ is the correction term for the BCC lattice, which can be approximated as

$$\triangle E_C = C_{\mathrm{BCC}} \frac{(Z_{\mathrm{non}} e)^2}{a}. \tag{55}$$

Here a is the lattice constant as defined in Eq. (44), the coefficient $C_{\mathrm{BCC}} = 0.006562$ is taken from Ref. [8], and Z_{non} is the non-uniform part of the charge number per cell given by

$$Z_{\mathrm{non}} = \int_0^{R_p} (n_p^{\mathrm{in}} - n_p^{\mathrm{out}} - 2n_\alpha^{\mathrm{out}}) \left[1 - \left(\frac{r}{R_p} \right)^{t_p} \right]^3 4\pi r^2 dr. \tag{56}$$

Because of the long-range nature of the Coulomb interaction, the Coulomb energy will depend on the lattice type. This dependence was discussed in more detail in Ref. [8]. The system prefers the BCC lattice because the BCC lattice gives the lowest Coulomb energy.

5   Calculation procedure in detail

We calculate the EOS table in the range mentioned in Sec. 1. For each T, Y_p, and ρ_B, we perform the minimization of the free energy for non-uniform matter using the Thomas-Fermi method described in Sec. 4. We also do the minimization for uniform matter with respect to converting two protons and two neutrons into an alpha-particle, in which there is only one independent parameter. Here, the phase of heavy nuclei formed together with free nucleons and alpha-particles is referred to as non-uniform matter, while the phase of nucleons mixed with alpha-particles without heavy nuclei is referred to as uniform matter. By comparing the free energy of non-uniform matter with that of uniform matter, we determine the most favorable state of nuclear matter at this T, Y_p, and ρ_B. The density of the phase transition between uniform matter and non-uniform matter depends on both T and Y_p. The non-uniform matter phase can exist only in the low temperature range ($T < 14$ MeV).

We construct the EOS table in three dimensions (T, Y_p, and ρ_B). We first fix T, and then fix Y_p and ρ_B. We finish the following steps in order to work out an EOS table with fixed T where the non-uniform matter phase exists.

- 1 calculate input table in the RMF theory for the use of minimization

- 2 do minimization for uniform matter at low density

- 3 do minimization for non-uniform matter at medium density

- 4 calculate the RMF result for uniform matter without alpha-particles at high density

- 5 determine the phase transition and connect the free energy

- 6 calculate the pressure and chemical potentials from the free energy

- 7 construct the finial EOS table at this T

Now, we explain each step in more detail.

- 1 The calculation of the minimization procedure needs the free energy per baryon as a function of n_n and n_p as input table, and this input table should be big enough in order to get good numerical accuracy in doing linear approximation. We calculate this input table in the RMF theory which can describe the homogeneous matter of protons and neutrons. This input table covers the following range

$Y_p = \frac{n_p}{n_n+n_p}$	from 0.0	to 0.5	mesh $= 0.0005$	point $= 1001$
$n_B = n_n + n_p \, (\text{fm}^{-3})$	from 0.000010	to 0.000100	mesh $= 0.000001$	point $= 91$
	from 0.000110	to 0.001000	mesh $= 0.000010$	point $= 90$
	from 0.001100	to 0.060000	mesh $= 0.000100$	point $= 590$
	from 0.061000	to 0.160000	mesh $= 0.001000$	point $= 100$

The density meshes are determined by the requirement to get good accuracy for the minimization results. In this input table, we list the quantities of the energy per baryon E and the entropy per baryon S. The free energy per baryon can be obtained by $F = E - TS$. We note that the data at $n_n + n_p > 0.16 \, \text{fm}^{-3}$ is not necessary for the minimization procedure, while the ideal-gas approximation is used to describe neutrons and protons when $n_n + n_p < 10^{-5} \, \text{fm}^{-3}$.

- 2 At low density and finite temperature, the thermodynamically favorable state might be a uniform matter which is a mixed gas of neutrons, protons, and alpha-particles. For each Y_p and ρ_B, we do minimization in order to find out the alpha-particle fraction having the minimum free energy. Since $n_B = n_n + n_p + 4n_\alpha$ and $Y_p = (n_p + 2n_\alpha)/n_B$ are fixed in this case, there is

only one independent parameter, the alpha-particle fraction $X_\alpha = 4n_\alpha/n_B$, in this minimization procedure.

• 3 The non-uniform matter phase exists in some middle density range where the heavy nuclei can be formed in order to lower the free energy. The starting density of non-uniform matter phase depends on the temperature strongly, while the ending density is nearly independent of the temperature. Both of them have a weak dependence on Y_p. We first try to find out the starting density at T, and then calculate the non-uniform matter results in this range by using the minimization codes with respect to some independent parameters as described in Sec. 4. Actually, we have to use a few codes to do this calculation because of the numerical difficulty in the minimization procedure. As the density ρ_B increases with fixed T and Y_p, the heavy nucleus fraction increases, while the free nucleon fractions and alpha-particle fraction decrease. When one of them decreases to a small value of about 10^{-5}, it brings difficulty in the minimization code, and lose good accuracy in the results. In this case, we use another code which takes this composition out by deleting one independent parameter. It may run into difficulty again as the density increases, and then we delete one more parameter. We list all codes used in this procedure in Table 2. We determine which code should be used by comparing the free energies, and choose the one with the lowest free energy. Generally speaking, the heavy nuclei use up free protons ($Y_p < 0.45$) or free neutrons ($Y_p > 0.45$) quickly after the non-uniform matter appears. The alpha-particles will disappear as the density increases, and then free neutrons ($Y_p < 0.45$) or free protons ($Y_p > 0.45$) disappear. Finally, only heavy nuclei are formed without free particles outside (this pure nucleus phase does not occur at $Y_p < 0.3$). When $T > 4$ MeV, the calculation becomes relatively easy because the favorable state is always the one including all compositions. As the temperature increases, the heavy nucleus fraction decreases, and it disappears completely when $T > 14$ MeV.

• 4 As the density ρ_B increases beyond $10^{14.2}$ g/cm³, the non-uniform matter phase disappears, and it becomes a homogeneous matter of neutrons and protons with less alpha-particles. The alpha-particle fraction X_α decreases as the density ρ_B increases. We switch off the alpha-particle degree of freedom when $X_\alpha < 10^{-4}$, and use the RMF code to calculate the results of uniform matter without alpha-particles in the high density range.

Table 2: The minimization codes for the non-uniform matter phase

code name	compositions	of non-uniform	matter		parameter number
npah	free neutrons,	free protons,	alpha particles,	heavy nuclei	8
pah		free protons,	alpha particles,	heavy nuclei	7
nah	free neutrons,		alpha particles,	heavy nuclei	7
nph	free neutrons,	free protons,		heavy nuclei	7
nh	free neutrons,			heavy nuclei	6
ph		free protons,		heavy nuclei	6
ah			alpha particles,	heavy nuclei	6
h				heavy nuclei	5

• 5 We determine the phase transitions in the whole density range at each Y_p with fixed T by comparing the free energies calculated with the uniform matter codes and non-uniform matter codes listed in Table 2. All the coming results in steps 2, 3, and 4 are the inputs for the judging code, and we get two important output files from it, EOS.FE and EOS.TAB. In EOS.FE, the free energy per particle F is written as a function of Y_p and ρ_B with fixed T, which can be used for calculating pressure and chemical potentials. In EOS.TAB, all quantities that are needed for the finial EOS table are included.

• 6 We calculate pressure and chemical potentials through the following thermodynamical relations

$$p = \left[n_B^2 (\partial F / \partial n_B) \right]_{T, Y_p}, \tag{57}$$

$$\mu_n = \left[\partial (n_B F) / \partial n_n \right]_{T, n_p}, \tag{58}$$

$$\mu_p = \left[\partial (n_B F) / \partial n_p \right]_{T, n_n}, \tag{59}$$

where F is the free energy per baryon written in EOS.FE obtained in step 5, while n_B is the baryon number density related to the baryon mass density as $\rho_B = m_u n_B$ with $m_u = 931.494$ MeV being the atomic mass unit [9]. $n_n = (1 - Y_p) n_B$ and $n_p = Y_p n_B$ are the neutron and proton number densities, respectively. The numerical derivatives are calculated by a five-point

differentiation method. In the high density range where alpha-particles have been switched off, we take the exact results calculated in the RMF theory instead of the numerical differentiations. The output files of this step are PRE.TAB for pressure and CHE.TAB for chemical potentials.

- 7 We combine the files EOS.TAB, PRE.TAB, and CHE.TAB in order to get the finial EOS table at this T. Due to the use of many codes listed in Table 2 for the non-uniform matter phase, some fractions might be equal to zero at some densities in the file EOS.TAB. This is because these compositions have been switched off for getting good accuracy. However, we know that the fraction should be a small value at finite temperature if the chemical potential is finite. In this case, we calculate the fraction X_i by the chemical potential μ_i and write it in the finial EOS table,

$$X_i = (n_i^{\text{out}} V^{\text{out}})/(n_B V_{\text{cell}}), \tag{60}$$

where V_{cell} is the cell volume as defined in Eq. (44), while $V^{\text{out}} = V_{\text{cell}} - \frac{4\pi}{3} R_A^3$ is the volume outside the heavy nucleus with R_A being the maximum of R_p and R_n considered as the boundary of the heavy nucleus. n_i^{out} is the free neutron number density ($i = n$) or free proton number density ($i = p$). For alpha-particles ($i = \alpha$), n_i^{out} should be four times of the alpha-particle number density outside the heavy nucleus, because there are two protons and two neutrons in an alpha-particle. The number densities outside the heavy nucleus can be obtained through the relations,

$$n_n^{\text{out}} = 2 \left(\frac{MT}{2\pi} \right)^{3/2} \exp(\frac{\mu_n}{T}), \tag{61}$$

$$n_p^{\text{out}} = 2 \left(\frac{MT}{2\pi} \right)^{3/2} \exp(\frac{\mu_p}{T}), \tag{62}$$

$$n_\alpha^{\text{out}} = 8 \left(\frac{MT}{2\pi} \right)^{3/2} \exp(\frac{\mu_\alpha + B_\alpha}{T}), \tag{63}$$

where $\mu_\alpha = 2\mu_n + 2\mu_p$ is based on the equilibrium condition. The finial EOS table at this T is the output file of this step.

We have to go through these steps for each T where the non-uniform matter phase exists. We also add the results for zero temperature case ($T = 0$). There are two main differences between $T = 0$ and $T \neq 0$ cases. (1) At $T = 0$, there are no alpha-particles and free protons (or free neutrons) outside the heavy nucleus, so we need not calculate so many codes listed in Table 2. The starting density of the non-uniform matter phase is below $10^5 \, \text{g/cm}^3$ at $T < 0.4 \, \text{MeV}$, so

we can skip step 2 in this case. (2) The ideal-gas approximation is not available at $T = 0$, so we use a function Ck_f^2 based on the Fermi-gas model to express the nonrelativistic kinetic energy at low density. Note that the free energy at $T = 0$ is equal to the internal energy, because the entropy is equal to zero in this case.

When the non-uniform matter phase disappears at high temperature, the calculation becomes relatively easy. We can skip step 3 which is the most complicated part in this calculation. As the temperature increases, the alpha-particle contribution gets less and less. We completely neglect the alpha-particle contribution at $T > 100\,\mathrm{MeV}$. On the other hand, the alpha-particle contribution is also dependent on the density. As the density ρ_B increases, the alpha-particle fraction X_α increases, but it drops rapidly at high density due to the finite volume effect of alpha-particles. In the high temperature range $14\,\mathrm{MeV} < \mathrm{T} < 100\,\mathrm{MeV}$, we take into account the alpha-particle contribution from $\rho_B = 10^{10}\,\mathrm{g/cm^3}$ to the density where X_α becomes smaller than 10^{-4}. If X_α is smaller than 10^{-4} in the whole density range, we neglect the alpha-particle contribution and calculate X_α by the chemical potential μ_α. The difference between the ideal-gas approximation and the RMF results is mainly due to the relativistic effect, and it increases as the temperature increases. We use the RMF theory to perform the calculation in the whole density range at $T > 30\,\mathrm{MeV}$.

At the end, we calculate the pure neutron matter results ($Y_p = 0$). There is only one composition in this case, so we can use the RMF theory to perform the calculation in the whole density range, and get the final EOS table for $Y_p = 0$.

6   Resulting EOS table

In this section, we explain the resulting EOS table and list the definitions of the physical quantities tabulated. We provide the resulting EOS by three tables, which are named as

(1) eos2.tab (main EOS table, size: 139.81 MB)

- temperature T [MeV]: $-1.0 \leq \log_{10}(T) \leq 2.6$; mesh of $\log_{10}(T) = 0.04$

- proton fraction Y_p: $0.01 \leq Y_p \leq 0.65$; mesh of $Y_p = 0.01$

- baryon mass density ρ_B [g/cm³]: $5.1 \leq \log_{10}(\rho_B) \leq 16.0$; mesh of $\log_{10}(\rho_B) = 0.1$

(2) eos2.t00 (EOS at $T = 0$, the same range of Y_p and ρ_B, size: 1.54 MB)

(3) eos2.yp0 (EOS at $Y_p = 0$, the same range of T and ρ_B, size: 2.18 MB)

One can download them from the following websites:

http://physics.nankai.edu.cn/grzy/shenhong/EOS/index.html

http://www.rcnp.osaka-u.ac.jp/~shen/

In the model used in this calculation, we consider four compositions which are free neutrons, free protons, alpha-particles, and a single species of heavy nuclei. Generally speaking, there are two phases existing in the range covered by this EOS table. The phase where heavy nuclei are formed is referred to as non-uniform matter, while the phase without heavy nuclei is referred to as uniform matter. The phase transition can be found by the variation of the heavy nucleus fraction between $X_A = 0$ and $X_A \neq 0$.

We write the EOS table in the following order: first fix T which is written at the beginning of each block in the table, second fix Y_p, third fix ρ_B. The blocks with different T are divided by the string of characters 'cccccccccccc'. For each T, Y_p, and ρ_B, we write all quantities in one line by the following order:

- (1) logarithm of baryon mass density: $\log_{10}(\rho_B)$ [g/cm^3]

- (2) baryon number density: n_B [fm^{-3}]

  The baryon number density is related to the baryon mass density as $\rho_B = m_u n_B$ with $m_u = 931.494$ MeV being the atomic mass unit taken from Ref. [9].

- (3) proton fraction: Y_p

  The proton fraction Y_p of uniform matter is defined by

$$Y_p = \frac{n_p + 2n_\alpha}{n_B} = \frac{n_p + 2n_\alpha}{n_n + n_p + 4n_\alpha}, \tag{64}$$

  where n_p is the proton number density, n_n is the neutron number density, n_α is the alpha-particle number density, and n_B is the baryon number density.

  For non-uniform matter case, Y_p is the average proton fraction defined by

$$Y_p = \frac{N_p}{N_B}, \tag{65}$$

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where N_p is the proton number per cell, and N_B is the baryon number per cell given by

$$N_p = \int_{\text{cell}} \left[n_p(r) + 2n_\alpha(r) \right] d^3r, \tag{66}$$

$$N_B = \int_{\text{cell}} \left[n_n(r) + n_p(r) + 4n_\alpha(r) \right] d^3r. \tag{67}$$

Here, $n_p(r)$ and $n_n(r)$ are the proton and neutron density distribution function given by Eq. (45), and $n_\alpha(r)$ is the alpha-particle density distribution function given by Eq. (47).

- (4) free energy per baryon: F [MeV]

  The free energy per baryon is defined as relative to the free nucleon mass $M = 938$ MeV in the TM1 parameter set as

  $$F = \frac{f}{n_B} - M. \tag{68}$$

- (5) internal energy per baryon: E_{int} [MeV]

  The internal energy per baryon is defined as relative to the atomic mass unit $m_u = 931.494$ MeV as

  $$E_{\text{int}} = \frac{\epsilon}{n_B} - m_u. \tag{69}$$

- (6) entropy per baryon: S [k_B]

  The entropy per baryon is related to the entropy density via

  $$S = \frac{s}{n_B}. \tag{70}$$

- (7) mass number of the heavy nucleus: A

  The mass number of the heavy nucleus is defined by

  $$A = \int_0^{R_A} \left[n_n(r) + n_p(r) \right] 4\pi r^2 dr, \tag{71}$$

  where $n_n(r)$ and $n_p(r)$ are the density distribution functions given by Eq. (45), and R_A is the maximum of R_p and R_n, which is considered as the boundary of the heavy nucleus.

- (8) charge number of the heavy nucleus: Z

  The charge number of the heavy nucleus is defined by

  $$Z = \int_0^{R_A} n_p(r) \, 4\pi r^2 dr. \tag{72}$$

- (9) effective nucleon mass: M^* [MeV]

  The effective nucleon mass is obtained in the RMF theory for uniform matter. In the non-uniform matter phase, the effective nucleon mass is a function of space due to inhomogeneity of the nucleon distribution, so it is meaningless to list the effective nucleon mass for non-uniform matter. We replace the effective nucleon mass M^* by the free nucleon mass M in the non-uniform matter phase.

- (10) free neutron fraction: X_n

  The free neutron fraction is given by

$$X_n = (n_n^{\text{out}} V^{\text{out}})/(n_B V_{\text{cell}}), \tag{73}$$

  where V_{cell} is the cell volume as defined in Eq. (44), $V^{\text{out}} = V_{\text{cell}} - \frac{4\pi}{3} R_A^3$ is the volume outside the heavy nucleus, n_n^{out} is the free neutron number density outside the heavy nucleus, and n_B is the average baryon number density.

- (11) free proton fraction: X_p

  The free proton fraction is given by

$$X_p = (n_p^{\text{out}} V^{\text{out}})/(n_B V_{\text{cell}}), \tag{74}$$

  where n_p^{out} is the free proton number density outside the heavy nucleus.

- (12) alpha-particle fraction: X_α

  The alpha-particle fraction is defined by

$$X_\alpha = 4N_\alpha/(n_B V_{\text{cell}}), \tag{75}$$

  where N_α is the alpha-particle number per cell obtained by

$$N_\alpha = \int_{\text{cell}} n_\alpha(r) \, d^3r, \tag{76}$$

  and $n_\alpha(r)$ is the alpha-particle density distribution function given by Eq. (47).

- (13) heavy nucleus fraction: X_A

  The heavy nucleus fraction is defined by

$$X_A = A/(n_B V_{\text{cell}}), \tag{77}$$

where A is the mass number of the heavy nucleus as defined in Eq. (71).

- (14) pressure: p [MeV/fm³]

  The pressure is calculated through the thermodynamical relation

$$p = \left[n_B^2 (\partial F / \partial n_B) \right]_{T, Y_p}. \tag{78}$$

- (15) chemical potential of the neutron: μ_n [MeV]

  The chemical potential of the neutron relative to the free nucleon mass M is calculated through the thermodynamical relation

$$\mu_n = \left[\partial (n_B F) / \partial n_n \right]_{T, n_p}. \tag{79}$$

Here $n_n = (1 - Y_p)\, n_B$.

- (16) chemical potential of the proton: μ_p [MeV]

  The chemical potential of the proton relative to the free nucleon mass M is calculated through the thermodynamical relation

$$\mu_p = \left[\partial (n_B F) / \partial n_p \right]_{T, n_n}. \tag{80}$$

Here $n_p = Y_p\, n_B$.

We have done the following check for the EOS table:

(1) the consistency of the fractions,

$$X_n + X_p + X_\alpha + X_A = 1. \tag{81}$$

(2) the consistency of the relation between F, E_{int}, and S,

$$F = E_{\text{int}} - TS + m_u - M. \tag{82}$$

(3) the consistency of the thermodynamical quantities,

$$F = \mu_n (1 - Y_p) + \mu_p Y_p - \frac{p}{n_B}. \tag{83}$$

In general, these consistency relations can be satisfied within a few thousandths. Physical constants to convert units are taken from Ref. [9].

7 Suggestions for using the EOS table

This relativistic EOS table is designed for use in supernova simulations, so we perform the calculation at each T, Y_p, and ρ_B. Comparing with EOS1 done in 1998, we have improved the numerical codes to take regular logarithmic meshes for $\log_{10}(T)$ and $\log_{10}(\rho_B)$, and use a linear mesh for Y_p in EOS2.

In order to perform the minimization of the free energy for non-uniform matter, we have to parameterize the nucleon distributions, so that some quantities in the EOS table like A, Z, X_n, X_p, X_α, and X_A are dependent on this parameterization method. One should keep in mind their definitions when these quantities are used. We suggest users pay more attention to the thermodynamical quantities like F, E_{int}, S, p, μ_n, and μ_p, which are supposed not to be sensitive to the parameterization method. The thermodynamical quantities are found to be more smooth in the resulting EOS, while the use of many codes in Table 2 may bring some fluctuations in the fractions, especially in the temperature range $1\,\mathrm{MeV} < \mathrm{T} < 4\,\mathrm{MeV}$, where the connection is very complicated.

We hope this EOS table can be used in your calculation successfully. We emphasize again that this EOS table includes only baryon contributions, so you have to add the lepton contributions when you use it.

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